

PHYSICS 2020

Homework #4

(Due October 7, 2026, 6:00 pm)

Show all work. Include appropriate units with all numerical answers. Give numerical answers to three or four significant figures unless otherwise indicated. Use $g = 9.8 \text{ m/s}^2$ unless otherwise specified.

1. A ball is launched horizontally from the top of a 25.0 m high building with an initial speed of 12.0 m/s. Neglect air resistance.

- (a) How long does the ball remain in the air?
- (b) How far horizontally from the base of the building does the ball strike the ground?
- (c) What is the vertical component of the ball's velocity just before it strikes the ground?

2. A projectile is launched from level ground with an initial speed of 30.0 m/s at an angle of 40.0° above the horizontal. Neglect air resistance.

- (a) Find the initial horizontal and vertical components of the velocity.
- (b) Find the maximum altitude reached by the projectile.
- (c) Find the total time of flight.
- (d) Find the range of the projectile.

3. A cannon fires a projectile from level ground with a muzzle velocity of 50.0 m/s. A target is located on level ground 180 m away.

- (a) Determine the two possible firing angles that will cause the projectile to strike the target.
- (b) Which trajectory has the longer time of flight?

4. The same cannon has a muzzle velocity of 50.0 m/s, but a target is now located on level ground 260 m away.

- (a) Attempt to determine the firing angle needed to strike the target.
- (b) Explain mathematically why there is no real firing angle that will strike the target.
- (c) What is the maximum possible range of the cannon?
- (d) By how much is the target beyond the maximum range?

5. A newly planted tree requires 30.0 gal of water twice per week. Rather than measuring the water one gallon at a time, you decide to calibrate a garden hose. You turn on the hose to the setting you plan to use for watering and find that it takes 12.0 s to fill a 1.00 gal bucket.

- (a) Find the volume flow rate Q of the hose in gallons per minute (gal/min).
- (b) For how many minutes must the hose run to deliver 30.0 gal of water to the tree?

6. Newton's method for finding a zero of a function $f(x)$ is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Use Newton's method to find a solution of

$$x^2 - 10 = 0.$$

Begin with $x_0 = 3.00$.

- (a) Write the Newton iteration formula for this particular function.
- (b) Carry out three iterations to find x_1 , x_2 , and x_3 .
- (c) Compare x_3 with the value of $\sqrt{10}$ given by a scientific calculator.

7. A cannon is located at the origin and fires a projectile with a fixed muzzle velocity of 40.0 m/s. A target on a hillside is located at

$$(x_t, y_t) = (100 \text{ m}, 20.0 \text{ m}).$$

For a projectile launched at an angle θ , its trajectory is

$$y = x \tan \theta - \frac{gx^2}{2v_0^2 \cos^2 \theta}.$$

Thus the required firing angle satisfies

$$f(\theta) = x_t \tan \theta - \frac{gx_t^2}{2v_0^2 \cos^2 \theta} - y_t = 0,$$

with

$$f'(\theta) = x_t \sec^2 \theta - \frac{gx_t^2}{v_0^2} \sec^2 \theta \tan \theta.$$

Use Newton's method,

$$\theta_{n+1} = \theta_n - \frac{f(\theta_n)}{f'(\theta_n)},$$

to find the lower firing angle that will strike the target. Begin with $\theta_0 = 30.0^\circ$ and continue until successive values of θ differ by less than 0.01° .

Important: When evaluating $f(\theta)$ and $f'(\theta)$ in the Newton iteration, angles must be treated in radians.

8. Find the Error. A student is trying to determine the angle at which a projectile must be launched to strike a target on level ground 120 m away. The muzzle velocity is 40.0 m/s.

The student writes

$$R = \frac{v_0^2 \sin(2\theta)}{g}$$

and therefore

$$\sin(2\theta) = \frac{Rg}{v_0^2} = \frac{(120)(9.8)}{(40.0)^2} = 0.735.$$

The student then calculates

$$2\theta = \sin^{-1}(0.735) = 47.3^\circ$$

and concludes that

$$\theta = 23.7^\circ.$$

The student reports that 23.7° is the only possible firing angle.

- Is 23.7° a valid firing angle?
- What error has the student made in concluding that this is the only possible angle?
- Determine the other firing angle that will strike the target.
- Explain why two different firing angles can give the same range when the projectile is launched and lands at the same altitude.