Weighted Averages

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Introduction

A computation that appears frequently in physics is the calculation of a *weighted average*. The idea is that you want to take the average of a collection of numbers, but you wish to assign a greater "weight", or importance, to some numbers, and a lesser weight to others. If you have a finite set of numbers f_i whose average you wish to compute, and a corresponding set of weights w_i for each number, then the weighted average is computed as the sum of the products of each number and its weight, divided by the sum of all the weights:

$$f_{\text{ave}} = \frac{\sum_i f_i \, w_i}{\sum_i w_i}. \tag{1}$$

This idea can be generalized to allow the calculation of the weighted average of a continuous function $f(x)$. Here the individual weights are replaced by a *weighting function $w(x)$*, so that the "weight" of $f(x)$ at x is $w(x) \, dx$. We also replace the summations in Eq. (1) with integrals; the weighted average of $f(x)$ is then defined as the ratio of two integrals,

$$f_{\text{ave}} = \frac{\int f(x) \, w(x) \, dx}{\int w(x) \, dx}, \tag{2}$$

where both integrals are over the same interval, and the result is the weighted average for that interval.

Examples

Center of Mass

We've already seen an example of this weighted average calculation: the center of mass. For simplicity, we'll consider here the calculation of the center of mass in one dimension. Given a set of (point) masses m_i, each of which is at position x_i, we find the center of mass as the weighted average of the positions, where the "weights" are the masses:

$$x_{\text{cm}} = \frac{\sum_i x_i \, m_i}{\sum_i m_i} \tag{3}$$

For the center of mass of a continuous body (i.e. a finite rod), the center of mass is computed as the weighted average of the positions occupied by the body, where the weighting function is the linear mass density function $\lambda(x)$:

$$x_{\text{cm}} = \frac{\int x \, \lambda(x) \, dx}{\int \lambda(x) \, dx}. \tag{4}$$

Course Grade

Another application of the weighted average is in the calculation of your course grade. For example, your PHY 1030 course grade is computed as:

Assignment	Symbol	Weight
Homework	H	0.15
Exam 1	E_1	0.20
Exam 2	E_2	0.20
Exam 3	E_3	0.20
Final	F	0.25

Let's look at some common questions related to the calculation of grades.

What Is My Final Grade?

The final grade is computed as a weighted average of the homework grade H, the exam grades E_1, E_2, and E_3, and the final exam grade F:

$$\text{final grade} \quad = \quad \frac{0.15H + 0.20E_1 + 0.20E_2 + 0.20E_3 + 0.25F}{0.15 + 0.20 + 0.20 + 0.20 + 0.25} \tag{5}$$

$$\tag{6}$$

$$= \quad 0.15H + 0.20E_1 + 0.20E_2 + 0.20E_3 + 0.25F. \tag{7}$$

For example, if you have a homework average $H = 92\%$, exam averages of $75\%, 80\%$, and 83%, and a final exam grade of 87%, then your final grade is computed as

$$(0.15)(92) + (0.20)(75) + (0.20)(80) + (0.20)(83) + (0.25)(87) = 83.15\%. \tag{8}$$

What Is My Grade Right Now?

If you want to know how you're doing in the class so far, simply compute the weighted average of the assignments completed so far. For example, if you know your (approximate) homework average and your grade for exams 1 and 2, but don't yet know your grade for exam 3 or the final, then your grade so far would be computed as a weighted average of the homework grade and the two exam grades:

$$\text{grade so far} \quad = \quad \frac{0.15H + 0.20E_1 + 0.20E_2}{0.15 + 0.20 + 0.20} \tag{9}$$

$$\tag{10}$$

$$= \quad \frac{0.15H + 0.20E_1 + 0.20E_2}{0.55}. \tag{11}$$

What Grade Do I Need to Get on the Final Exam?

Another common question is: how well do I need to do on the final exam to get an A for the course (or a B, or a C, or just to pass)? To answer this question, let's assume that we know the homework grade and all three exam grades. Then we substitute the goal percentage for the final grade on the left-hand side of the Eq. (7). We then find that, for example, to get an A in the class,

$$90 = 0.15H + 0.20E_1 + 0.20E_2 + 0.20E_3 + 0.25F. \tag{12}$$

Solving for the final exam grade F, we find the final exam grade required for an A in the course is

$$F = \frac{90 - 0.15H - 0.20E_1 - 0.20E_2 - 0.20E_3}{0.25}. \tag{13}$$

Similarly, the final exam grade needed to receive a B is found by replacing 90 with 80:

$$F = \frac{80 - 0.15H - 0.20E_1 - 0.20E_2 - 0.20E_3}{0.25}, \tag{14}$$

and to get a C, the required final exam grade is

$$F = \frac{70 - 0.15H - 0.20E_1 - 0.20E_2 - 0.20E_3}{0.25}, \tag{15}$$

and to just get a passing grade of D, the required final exam grade is

$$F = \frac{60 - 0.15H - 0.20E_1 - 0.20E_2 - 0.20E_3}{0.25}. \tag{16}$$

(A negative or zero result means that you've already earned *at least* that final grade for the course.)

For example, suppose you have a homework average $H = 94\%$, and exam averages of 85%, 90%, and 92%. What grade to you need to get on the final exam to get an A for the course?

$$F = \frac{90 - 0.15H - 0.20E_1 - 0.20E_2 - 0.20E_3}{0.25} \tag{17}$$

$$\tag{18}$$

$$= \frac{90 - (0.15)(94) - (0.20)(85) - (0.20)(90) - (0.20)(92)}{0.25} \tag{19}$$

$$\tag{20}$$

$$= 90\% \tag{21}$$

You should be able to use this same method for other courses, but it will have to be modified by replacing Eq. (7) with the equation your instructor uses to compute the final grade. Also, another instructor may have different criteria for each letter grade (not necessarily 90%=A, 80%=B, etc.).