Trigonometric Identities

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Basic Formulæ

$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$\sec^2\theta \equiv 1 + \tan^2\theta$$
$$\csc^2\theta \equiv 1 + \cot^2\theta$$

Angle Addition Formulæ

$$\sin(\alpha \pm \beta) \equiv \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) \equiv \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) \equiv \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Double-Angle Formulæ

$$\sin 2\theta \equiv 2\sin\theta\cos\theta$$
$$\cos 2\theta \equiv \cos^2\theta - \sin^2\theta \equiv 1 - 2\sin^2\theta \equiv 2\cos^2\theta - 1$$
$$\tan 2\theta \equiv \frac{2\tan\theta}{1 - \tan^2\theta}$$

Half-Angle Formulæ

$$\sin\frac{\theta}{2} \equiv \pm\sqrt{\frac{1 - \cos\theta}{2}}$$
$$\cos\frac{\theta}{2} \equiv \pm\sqrt{\frac{1 + \cos\theta}{2}}$$
$$\tan\frac{\theta}{2} \equiv \frac{\sin\theta}{1 + \cos\theta} \equiv \frac{1 - \cos\theta}{\sin\theta}$$

Products of Sines and Cosines

$$\sin\alpha\ \cos\beta \equiv \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$$
$$\cos\alpha\ \sin\beta \equiv \frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$$
$$\cos\alpha\ \cos\beta \equiv \frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$$
$$\sin\alpha\ \sin\beta \equiv -\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]$$

Sums and Differences of Sines and Cosines

$$\sin\alpha+\sin\beta \equiv 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\sin\alpha-\sin\beta \equiv 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$
$$\cos\alpha+\cos\beta \equiv 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$\cos\alpha-\cos\beta \equiv -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

Other Formulæ

$$\sin^2\theta \equiv \frac{1}{2}\left(1-\cos 2\theta\right)$$
$$\cos^2\theta \equiv \frac{1}{2}\left(1+\cos 2\theta\right)$$
$$\tan\theta \equiv \cot\theta - 2\cot 2\theta$$