

Projectile Motion

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This table gives formulæ for a projectile initially at the origin, fired with initial velocity v_0 at an angle θ_0 from the horizontal.

Quantity	Formula
$x(t)$	$x = (v_0 \cos \theta_0)t$
$y(t)$	$y = -\frac{1}{2}gt^2 + (v_0 \sin \theta_0)t$
$y(x)$	$y(x) = \left(-\frac{g}{2v_0^2 \cos^2 \theta_0}\right)x^2 + (\tan \theta_0)x$
Time in flight	$t_f = \frac{2}{g}v_0 \sin \theta_0$
Range at angle θ_0	$R = \frac{v_0^2}{g} \sin 2\theta_0$
Max. range (at $\theta_0 = 45^\circ$)	$R_{\max} = \frac{v_0^2}{g}$
Angle needed to hit target at range R for fixed v_0	$\theta_0 = \frac{1}{2} \sin^{-1} \left(\frac{gR}{v_0^2} \right)$
Speed needed to hit target at range R for fixed θ_0	$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}}$
Max. altitude	$h = \frac{v_0^2 \sin^2 \theta_0}{2g}$
Speed needed to hit target at (x_t, y_t) for fixed θ_0	$v_0 = \sqrt{\frac{gx_t}{2\left(\tan \theta_0 - \frac{y_t}{x_t}\right) \cos^2 \theta_0}}$
Angle needed to hit target at (x_t, y_t) for fixed v_0	$x_t \sin 2\theta_0 - 2y_t \cos^2 \theta_0 = \frac{gx_t^2}{v_0^2}$

Note that this last expression must be solved iteratively for θ_0 .