

# Photometry

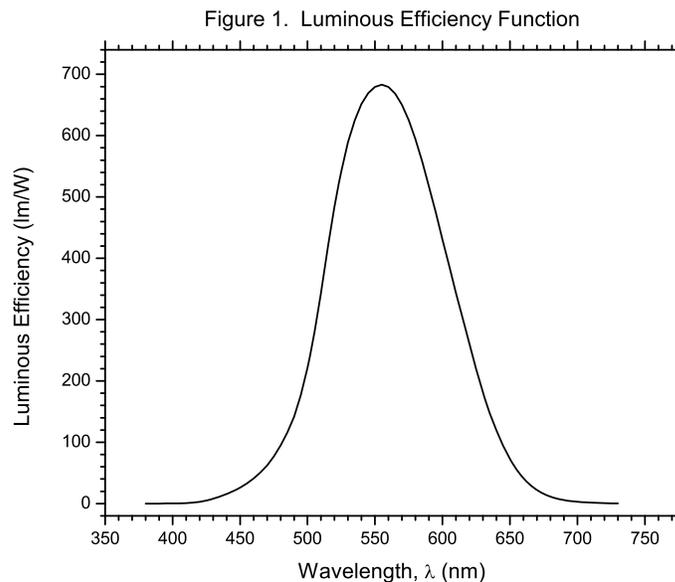
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May 10, 2010

A frequently neglected area of optics is the field of *photometry*—the study of the measurement of the brightness of light. A related field is *radiometry*, where one measures the intensity of electromagnetic radiation at all wavelengths. In photometry, though, we take into account the physiology of human vision. The goal of photometry is to measure the brightness of visible light, *as it appears to the human eye*.

We begin with a simple mathematical model of human vision. Figure 1 shows such a model, called the *luminous efficiency curve*; it models how the eye's sensitivity varies with wavelength. As shown by the figure, the human eye is most sensitive to visible light in the green part of the spectrum, at a wavelength of about 555 nm. The eye is much less sensitive to red and violet light, where the curve has values near zero.



## 1 Luminous Flux

We now introduce definitions of some basic photometric quantities. First, the *luminous flux*  $\Phi$  is the total amount of visible light emitted by a light source, in all directions. Luminous flux is analogous to the total amount of electromagnetic radiation emitted by the light source, except that it is “weighted” by the luminous efficiency curve. For example, electromagnetic radiation with a wavelength near 555 nm is given more “importance” than radiation with a wavelength near 400 nm. This weighted average is luminous flux.

In SI units, luminous flux is measured in units of *lumens* (lm). If you look closely at Figure 1, you’ll see that the vertical axis has units of lumens per watt (lm/W). When we take the intensity of electromagnetic radiation (in watts) and multiply by this luminous efficiency curve to “weight” different wavelength according to the sensitivity of human vision, we get units of lumens. Note that the peak of the luminous efficiency curve is at  $\lambda = 555$  nm, where the human eye is most sensitive; at this wavelength the luminous efficiency is 683 lm/W.

You may see the lumen used on packages of light bulbs, where it may be listed as the “light output”. For example, a typical 60-watt incandescent light bulb may have a luminous flux of 820 lumens. This means that the bulb consumes electric power at the rate of 60 watts (60 joules of energy per second), while producing 820 lumens of light. High-efficiency light bulbs produce more visible light while using less electric power, at the expense of producing less electromagnetic radiation at non-visible wavelengths. For example, a compact fluorescent light bulb may produce 1200 lumens of light, while consuming only 20 watts of electric power. If you’re trying to replace incandescent light bulbs with compact fluorescent bulbs, you should try to find a compact fluorescent bulb that has a luminous flux (in lumens) similar to that of the bulb you’re replacing. *Don’t* replace it with a bulb that has the same power consumption (in watts). For example, a 60-watt incandescent bulb that emits 820 lumens of light should be replaced by a compact fluorescent bulb that emits about 820 lumens of light.

## 2 Luminous Intensity

A quantity related to luminous flux is *luminous intensity*  $I$ , which is the luminous flux  $\Phi$  per unit solid angle  $\Omega$ :

$$I = \frac{\Phi}{\Omega} \quad (1)$$

The SI unit of luminous intensity is the *candela* (cd): one candela is equal to one lumen per steradian. A candela is approximately equal an older unit called the *candlepower*, which was the light intensity emitted by the flame of a candle. So a candle flame has a luminous intensity of about 1 candela; by comparison, a 60-watt incandescent light bulb has a luminous intensity of about 65 candelas, while a typical searchlight has a luminous intensity of about 800 million candelas.

If a light source is *isotropic* (so it emits light equally in all directions), then there is a simple relationship between luminous flux  $\Phi$  and luminous intensity  $I$ :  $I = \Phi / (4\pi \text{ sr})$ .

The candela is the fundamental photometric unit in SI units, and is determined as the result of an experiment. Other photometric units (the lumen and the lux) are defined in terms of the candela.

### 3 Illuminance

The level of illumination seen by an observer is called the *illuminance*. To find the illuminance  $E$ , we divide the luminous flux  $\Phi$  emitted by the light source by the area  $A$  over which that luminous flux is spread:

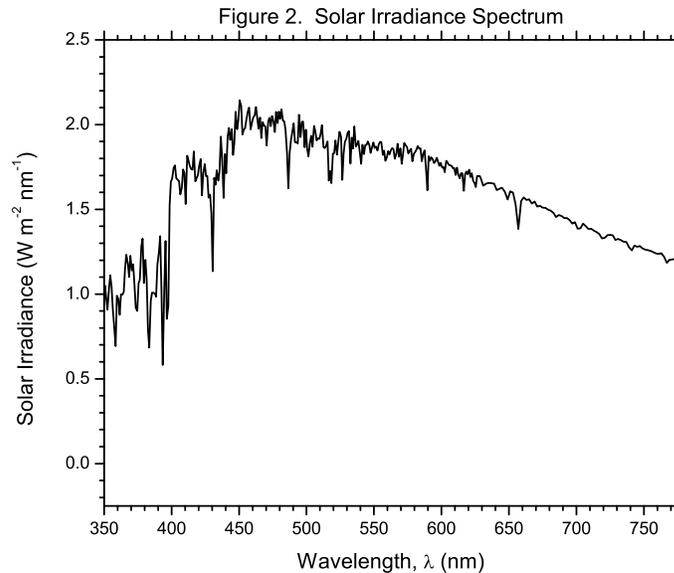
$$E = \frac{\Phi}{A} \quad (2)$$

The SI unit of illuminance is the *lux* (lx), where one lux is one lumen per square meter. To give a sense of scale, the level of illumination in a typical office is around 400 lux, while direct sunlight is around 100,000 lux (depending on how high the Sun is in the sky, cloud conditions, etc.).

(An older unit of illuminance, the *foot-candle*, is one lumen per square foot, or about 10.76391 lux.)

### 4 Example: The Sun

As an example of how photometric calculations are done, consider the Sun. To find the illuminance of the Sun at the Earth, we would begin by measuring the intensity of the Sun's radiation at different wavelengths; this gives a plot of the *solar irradiance spectrum*. Fig. 2 shows just part of the solar irradiance spectrum—the part that's within visible light wavelengths. We multiply this solar spectrum by the luminous efficiency curve (Fig. 1), and find the area under the resulting curve. The result is the illuminance of the Sun's light at the Earth, and works out to be  $E = 133,000$  lux.



We can now use Eq. (2) to find the luminous flux of the Sun. Here  $A$  is the total area over which the luminous flux is spread to give illuminance  $E$ , so  $A = 4\pi r^2$ , where

$r$  is the distance of the Earth from the Sun. We find

$$\Phi = EA \quad (3)$$

$$= E(4\pi r^2) \quad (4)$$

$$= (133,000 \text{ lux})[4\pi(1.4959787 \times 10^{11} \text{ m})^2] \quad (5)$$

$$= 3.75 \times 10^{28} \text{ lumens.} \quad (6)$$

From this result, we can compute the luminous intensity of the Sun. Since the Sun emits light equally in all directions (is isotropic), the luminous intensity  $I$  of the Sun is

$$I = \frac{\Phi}{\Omega} \quad (7)$$

$$= \frac{3.75 \times 10^{28} \text{ lm}}{4\pi \text{ sr}} \quad (8)$$

$$= 2.98 \times 10^{27} \text{ candelas.} \quad (9)$$

In summary, for the Sun, we find

- Luminous flux:  $\Phi = 3.75 \times 10^{28} \text{ lm}$
- Luminous intensity:  $I = 2.98 \times 10^{27} \text{ cd}$
- Illuminance at Earth:  $E = 133 \text{ klx}$

## 5 Example: Incandescent Light Bulb

A 60-watt incandescent light bulb emits a luminous flux of 820 lumens. If this light bulb is isotropic and is the only illumination in a room, then what is the illuminance at a distance of 80 cm from the light bulb?

*Solution.* From equation (2), the illuminance  $E$  is

$$E = \frac{\Phi}{A} \quad (10)$$

$$= \frac{\Phi}{4\pi r^2} \quad (11)$$

$$= \frac{820 \text{ lm}}{4\pi(0.80 \text{ m})^2} \quad (12)$$

$$= 102 \text{ lx} \quad (13)$$