Kepler's Laws

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Introduction

Kepler's Laws of planetary motion were derived by the German astronomer Johannes Kepler in the early 17th century, based on astronomical observations made by the Danish astronomer Tycho Brahe. The describe some of the basic motion of planets orbit the Sun (although they apply more generally to any two-body orbit problem).

1 Kepler's First Law

Each of the planets orbits the Sun in an elliptical orbit, with the Sun at one of the foci of the ellipse.

Before Kepler's time, it was assumed that the planets moved around the Sun in circles (or circles orbiting on circles), but the predictions failed to satisfactorily match observations. Kepler was the first to recognize that the planets did not move in circles, but in *ellipses*.

One can derive the equation of the orbital ellipse in plane polar coordinates, in the plane of the orbit. The result is

$$r = \frac{a(1-e^2)}{1 + e\cos(\theta - \omega)}. \tag{1}$$

Here (r, θ) are the plane polar coordinates of the planet, a is the semi-major axis of the orbit, e is the eccentricity of the orbit, and ω is the argument of perihelion.

2 Kepler's Second Law

A line drawn from the Sun to a planet sweeps out equal areas in equal times.

The essence of this law is that planets move more slowly when they're farther from the Sun, and speed up as they get closer to the Sun. A comet in a highly elliptical orbit will spend most of its time far from the Sun, moving very slowly; as it gets close to the Sun, it will speed up, quickly whip around the Sun, and then move away again.

Quantitatively, the area per unit time swept out by a line joining the Sun to a planet is given by

$$\frac{dA}{dt} = \frac{1}{2}\sqrt{GMa(1-e^2)}, \tag{2}$$

where A is the area, G is the universal gravitational constant, and M is the mass of the Sun. Since everything on the right-hand side of this equation is a constant, it follows that dA/dt is constant.

3 Kepler's Third Law

The square of the period of the orbit is proportional to the cube of the semi-major axis.

This law relates the period of a planet's orbit (i.e. the time required to complete one orbit) to its distance from the Sun. Mathematically, this law is expressed as

$$P^2 \propto a^3, \tag{3}$$

where P is the period of the orbit. The proportionality constant turns out to be $4\pi^2/GM$, so Kepler's Third Law becomes

$$P^2 = \frac{4\pi^2}{GM}\, a^3. \tag{4}$$