Physics Recreations:
The Coronavirus and Social Distancing

Dr. D.G. Simpson
Department of Natural Sciences
Prince George's Community College

March 25, 2020

1 The COVID-19 Coronavirus

The year 2020 has brought upon the Earth a world-wide pandemic, the coronavirus COVID-19. To fight the pandemic, we're given some instructions: wash your hands frequently (for at least 20 seconds); stay home if you're feeling sick; and avoid being around other people as much as possible (keep at least six feet between you and the nearest person). This last instruction has been called "social distancing." How does this help?

We're also told that following these instructions will help to "flatten the curve." What does this mean, and why is it important?

Christian Hubbs [1] has described how we can answer these questions by employing a mathematical model of the spread of a disease like COVID-19, using mathematical tools of the kind we use to model phenomena in physics. This field is called *mathematical epidemeology.* Several such mathematical models are available; Hubbs uses one called the SEIR model (for Susceptible – Exposed – Infectious – Recovered), which is an extension of the 1927 Kermack-McKendrick model [2]. In the SEIR model, we define four functions:

- $S(t)$ is the number of susceptible individuals as a function of time;
- $E(t)$ is the number of exposed individuals as a function of time;
- $I(t)$ is the number of infected individuals as a function of time;
- $R(t)$ is the number of "removed" individuals as a function of time; these are the people who can no longer be infected, either because they have recovered and are now immune, or because they have succumbed to the virus and are out of the population.

We also define a set of parameters (all with units of days^{-1}):

- α is the reciprocal of the incubation period;
- β is the average contact rate in the population;
- γ is the reciprocal of the mean infectious period.

Closely related to these is a dimensionless parameter called R_0, which represents how quickly the disease spreads. It is related to the above parameters through

$$R_0 = \frac{\beta}{\gamma} \tag{1}$$

The SEIR model characterizes the growth of the disease using a set of coupled first-order differential equations:

$$\frac{dS}{dt} = -\beta SI \tag{2}$$

$$\frac{dE}{dt} = \beta SI - \alpha E \tag{3}$$

$$\frac{dI}{dt} = \alpha E - \gamma I \tag{4}$$

$$\frac{dR}{dt} = \gamma I \tag{5}$$

$$N = S + E + I + R \tag{6}$$

The last equation is a constraint: the total population N is constant, so there are no births or migrations into the population.

To solve these differential equations on a computer, we can approximate the differentials as finite differences:

$$\frac{\Delta S}{\Delta t} = -\beta SI \tag{7}$$

$$\frac{\Delta E}{\Delta t} = \beta SI - \alpha E \tag{8}$$

$$\frac{\Delta I}{\Delta t} = \alpha E - \gamma I \tag{9}$$

$$\frac{\Delta R}{\Delta t} = \gamma I \tag{10}$$

$$N = S + E + I + R \tag{11}$$

Here, for example, ΔS is the change in the susceptible population S during a time increment Δt, etc. Hubbs solves these equations using a simple Euler method integrator, which appears to be adequate for this application.

2 Modeling COVID-19

To model the spread of the COVID-19 virus, we'll need values of the parameters α, β, and γ. Recent research on COVID-19 [3,4] has given some values for these parameters:

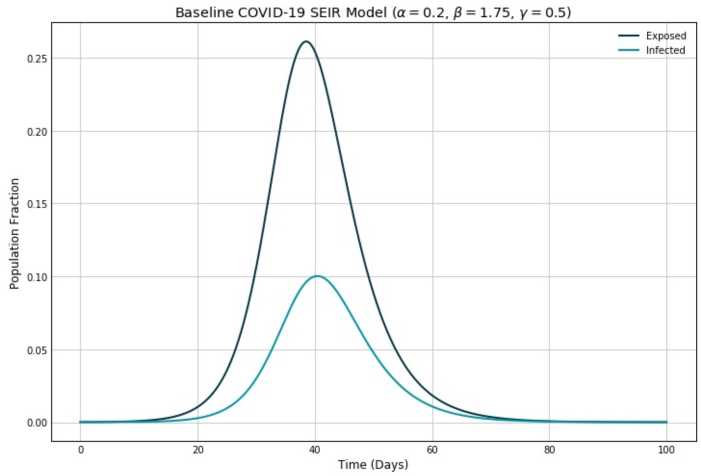

Figure 1: Modeled spread of the COVID-19 coronavirus, assuming no social distancing. At the peak of the run of the disease, 10% of the population is infected. (Figure from C. Hubbs, Ref. [1])

- Incubation period: 5 days, so that $\alpha = 1/(5 \text{ days}) = 0.2 \text{ day}^{-1}$.
- Mean infectious period: 2 days, so that $\gamma = 1/(2 \text{ days}) = 0.5 \text{ day}^{-1}$.
- Disease spread rate: $R_0 = \beta/\gamma = 3.5$, so that $\beta = \gamma R_0 = 1.75 \text{ day}^{-1}$.

We now have almost all we need to model the spread of the virus (that is, to plot $S(t)$, $E(t)$, $I(t)$, and $R(t)$) to get an idea of the intensity and duration of the pandemic. The only missing information is the initial conditions—that is, the values of S, E, I, and R at time $t = 0$. Let's assume, for example, that we have a population of $N = 10{,}000$, with initially one person exposed to the virus and the rest susceptible:

- $N = 10{,}000$
- $S(0) = 1 - 1/N$
- $E(0) = 1/N$
- $I(0) = 0$
- $R(0) = 0$

(Here, for numerical reasons, we scale S, E, I, and R by N, so that they represent the *fraction* of the population, rather than the actual number of individuals.) We can use a computer program to integrate the equations simultaneously; the results (which, so far, neglect social distancing) are shown in Figure 1.

3 Adding Social Distancing

So far, we have not tried to model the effects of social distancing. We can do this by introducing a social distancing parameter ρ ($0 \leq \rho \leq 1$), which varies from $\rho = 0$ for

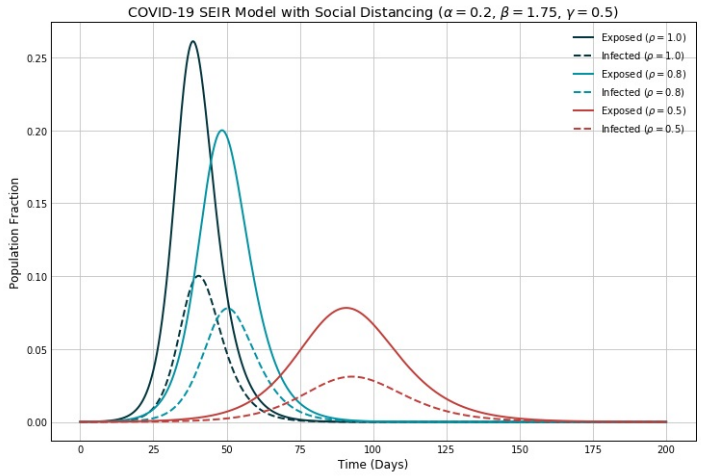

Figure 2: Modeled spread of the COVID-19 coronavirus, with social distancing. The social distancing parameter ρ is 1 for no social distancing; smaller values of ρ indicate increased social distancing, down to 0 for complete lockdown and isolation. Note that with increased social distancing, the duration of the pandemic is increased, in exchange for a decrease in the peak intensity. (Figure from C. Hubbs, Ref. [1])

complete lockdown (no contact between people) to $\rho = 1$ for no social distancing. In this case, we modify the first two of our equations thusly:

$$\frac{dS}{dt} = -\rho\beta SI \tag{2a}$$

$$\frac{dE}{dt} = \rho\beta SI - \alpha E \tag{3a}$$

Using finite differences, we approximate this as

$$\frac{\Delta S}{\Delta t} = -\rho\beta SI \tag{7a}$$

$$\frac{\Delta E}{\Delta t} = \rho\beta SI - \alpha E \tag{8a}$$

Integrating Eqs. (7a), (8a), (9), and (10) using an Euler method integrator for various values of ρ gives the plot in Figure 2.

From the figure, we can see what is meant by "flattening the curve"—the idea is to, by means of social distancing, extend the duration of the disease, in order to lower its peak intensity to a manageable level. In this model, with no social distancing ($\rho = 1$), the disease is shorter in duration, but we get a large peak of 10% of the population infected, which is likely to overwhelm the ability of medical facilities to deal with the crisis. On the other hand, with improved social distancing (say $\rho = 0.5$), the disease lasts longer, but only 3% of the population is infected at the peak, so that there's a better chance of providing medical care for everyone infected. Further simulations will show that the better the social distancing, the more manageable the pandemic.

4 References

This paper is based largely on Ref. [1], which includes some additional details, as well as Python code for implementing the computer models of the spread of the virus.

[1] Christian Hubbs, *Social Distancing to Slow the Coronavirus: Modeling the Flattening of the COVID-19 Peaks*, at URL:
`https://towardsdatascience.com/social-distancing-to-slow-the-coronavirus-768292f04296`
(Retrieved March 25, 2020.)

[2] William Ogilvy Kermack, A. G. McKendrick and Gilbert Thomas Walker. A contribution to the mathematical theory of epidemics, *Proc. R. Soc. Lond.*, A115700721 (1927).

[3] Joel Hellewell, Sam Abbott, Amy Gimma, *et al.*, Feasibility of controlling COVID-19 outbreaks by isolation of cases and contacts, *The Lancet Global Health* (2020) (`http://www.thelancet.com`),
doi:https://doi.org/10.1016/S2214-109X(20)30074-7

[4] Liangrong Peng, Wuyue Yang, Dongyan Zhang, *et al.*, Epidemic analysis of COVID-19 in China by dynamical modeling, arXiv:2002.06563v1 (2020).