Physics Recreations:
Music

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Music is a sequence of sounds created for enjoyment or artistic expression. The sounds may be produced by the human voice (singing), or by any number of musical instruments. Music is a vast field, and we can only hope to touch on some of the most basic ideas of music theory here; the interested reader is referred to books on music theory for more information.

1 Pitch

To begin, music consists of a sequence of sounds of short duration (called *notes*); each of these notes is at a specific frequency (called *pitch*). Not just any frequencies are used, though; musical notes are selected from a set of discrete frequencies.

We find that if we hear a sound at frequency f, then to our ears a sound at twice that frequency ($2f$) sounds "similar", but higher. To get musical notes, the interval between frequency f and $2f$, known as an *octave*, is divided into twelve equal parts (in a logarithmic sense) so that each note is higher in frequency than the next lower note by a factor of $\sqrt[12]{2} \approx 1.059463$. Each factor of $\sqrt[12]{2}$ change in frequency is called a *half step*, and two half steps make a *whole step*. The complete set of 12 notes in an octave (each separated in pitch by a half step) is called the *chromatic scale*.

Early musicians discovered that musical compositions sounded better when they used only certain subsets of these 12 notes, rather than all 12. One of the best-known of these subsets (or *scales*) consists of 7 of the 12 notes in an octave; these notes were named (in order of increasing pitch) C, D, E, F, G, A, and B. In this scale, called the *C major scale*, notes B and C (of the next octave) are one half step apart in frequency, as are notes E and F; the others are a whole step apart.

Each octave contains the 12 notes in the chromatic scale, which are given the following names, in order of increasing pitch:

Table 1. The musical notes.

C	F♯ / G♭
C♯ / D♭	G
D	G♯ / A♭
D♯ / E♭	A
E	A♯ / B♭
F	B

In this table we find the seven notes of the C major scale, along with the remaining five notes, which are named using the symbols ♯ and ♭ to indicate that they fall in between the notes of the C major scale. The symbol ♯ (called "sharp") indicates a raising in pitch by one half step over the note to which it is attached; similarly, the symbol ♭ (called "flat") indicates a lowering of pitch by one half step. For example, C♯ is one half step higher in pitch than C, and B♭ is one half step lower in pitch than B. (The symbols ♯ and ♭ are collectively called *accidentals*.)

Notice that several notes are known by two equivalent names. For example, C♯ and D♭ refer to the same note—the one between notes C and D. Also, since notes B and C are separated by just one half step, we have B♯ = C and C♭ = B; similarly, E and F are separated by one half step, so E♯ = F and F♭ = E.

When it is necessary to specify a specific octave, it is written as a subscript after the note. The note A_4 (near the middle of the piano keyboard) is assigned a frequency of exactly 440 Hz. Since the notes in each octave have twice the frequency of the same note in the next lower octave, we find the frequencies of note A in higher octaves by repeatedly multiplying by 2: $A_5 = 880$ Hz, $A_6 = 1760$ Hz, $A_7 = 3520$ Hz, $A_8 = 7040$ Hz, and $A_9 = 14080$ Hz. Similarly for A in lower octaves, we repeatedly divide 440 Hz by 2: $A_3 = 220$ Hz, $A_2 = 110$ Hz, $A_1 = 55$ Hz, and $A_0 = 27.5$ Hz. Human hearing covers ten octaves in pitch, going roughly from note E_0 to E_{10}. The piano's range is 7¼ octaves, from A_0 to C_8.

Beginning with the frequency of note $A_4 = 440$ Hz, we successively multiply and divide by $\sqrt[12]{2}$ to find the frequencies of all the other notes, as shown in Table 2.

Table 2. Frequencies (in hertz) of all the musical notes that are audible to the human ear. Middle C is shown in bold, and the musical standard A_4 is shown in italics.

	Octave										
Note	0	1	2	3	4	5	6	7	8	9	10
C		32.70	65.41	130.81	**261.63**	523.25	1046.50	2093.00	4186.01	8372.02	16744.04
C♯ / D♭		34.65	69.30	138.59	277.18	554.37	1108.73	2217.46	4434.92	8869.84	17739.69
D		36.71	73.42	146.83	293.66	587.33	1174.66	2349.32	4698.64	9397.27	18794.55
D♯ / E♭		38.89	77.78	155.56	311.13	622.25	1244.51	2489.02	4978.03	9956.06	19912.13
E	20.60	41.20	82.41	164.81	329.63	659.26	1318.51	2637.02	5274.04	10548.08	21096.16
F	21.83	43.65	87.31	174.61	349.23	698.46	1396.91	2793.83	5587.65	11175.30	
F♯ / G♭	23.12	46.25	92.50	185.00	369.99	739.99	1479.98	2959.96	5919.91	11839.82	
G	24.50	49.00	98.00	196.00	392.00	783.99	1567.98	3135.96	6271.93	12543.85	
G♯ / A♭	25.96	51.91	103.83	207.65	415.30	830.61	1661.22	3322.44	6644.88	13289.75	
A	27.50	55.00	110.00	220.00	*440.00*	880.00	1760.00	3520.00	7040.00	14080.00	
A♯ / B♭	29.14	58.27	116.54	233.08	466.16	932.33	1864.66	3729.31	7458.62	14917.24	
B	30.87	61.74	123.47	246.94	493.88	987.77	1975.53	3951.07	7902.13	15804.27	

In general, a note n half steps above A_4 has a frequency of

$$2^{n/12} \times 440 \text{ Hz}, \tag{1}$$

where n is negative for notes below A_4.

Note C_4 (in the middle of the piano keyboard) is called *middle C*. Since it's 9 half steps below A_4, middle C has a frequency of $2^{-9/12} \times 440 \text{ Hz} = 261.6256 \text{ Hz}$.

2 Musical Scales

As mentioned earlier, early musicians discovered that musical compositions sound best when they don't use all 12 notes of the chromatic scale; instead, restricting the notes to certain subsets of the 12 (called *scales*) results in more pleasant-sounding music.

In Western music, the most common of these scales are called *major scales*, and the best-known of these is the *C major scale*, which has already been described: it consists of the notes C, D, E, F, G, A, and B. In this scale, the first two notes (C and D) are separated in pitch by a whole step, as are the second and third notes (D and E). The third and fourth notes are separated by a half step. Continuing through the whole scale, we find that the separations between the notes in pitch are two whole steps, then one half step, then three whole steps, then another half step at the end when going from B to C of the next octave. For shorthand, let's write "W" for a whole-step interval between notes, and "H" for a half-step interval; then the intervals between notes in the C major scale can be written as WWHWWWH.

There are 11 other major scales besides the C major scale. To get them, we simply start with a different note in the chromatic scale, then follow the same WWHWWWH interval pattern; the scale is named for the note we started with. For example, for the C♯ major scale, we begin with C♯, then go up a whole step in pitch to get the next note in the scale, D♯. Then we go up another whole step to get the next note, F. Then up a half step to get the next note (F♯), and so on until we find all seven notes in the scale. Similarly, for the D major scale, we start with the note D and follow the same WWHWWWH pattern to find the seven notes of the D major scale. We can repeat the process for all 12 notes in the chromatic scale; the results are shown in Table 3.

Table 3. The major scales. The last column shows the number of accidentals in that scale.

Major Scale	Notes							# Acc.
C	C	D	E	F	G	A	B	0
G	G	A	B	C	D	E	F♯	1 ♯
D	D	E	F♯	G	A	B	C♯	2 ♯
A	A	B	C♯	D	E	F♯	G♯	3 ♯
E	E	F♯	G♯	A	B	C♯	D♯	4 ♯
B (=C♭)	B	C♯	D♯	E	F♯	G♯	A♯	5 ♯
F♯ (=G♭)	F♯	G♯	A♯	B	C♯	D♯	E♯ (=F)	6 ♯
C♯ (=D♭)	C♯	D♯	E♯ (=F)	F♯	G♯	A♯	B♯ (=C)	7 ♯
F	F	G	A	B♭	C	D	E	1 ♭
B♭	B♭	C	D	E♭	F	G	A	2 ♭
E♭	E♭	F	G	A♭	B♭	C	D	3 ♭
A♭	A♭	B♭	C	D♭	E♭	F	G	4 ♭
D♭ (=C♯)	D♭	E♭	F	G♭	A♭	B♭	C	5 ♭
G♭ (=F♯)	G♭	A♭	B♭	C♭ (=B)	D♭	E♭	F	6 ♭
C♭ (=B)	C♭ (=B)	D♭	E♭	F♭ (=E)	G♭	A♭	B♭	7 ♭

Notice that 15 scales are listed in this table; several of them (such as B and C♭) are really the same scale, but with the notes "spelled" differently (recall that some notes have two names, such as A♯=B♭), so there are really only 12 different major scales, each one beginning with a different note in the chromatic scale and following the WWHWWWH pattern.

Notice also in Table 3 that each major scale can be uniquely identified by the total number of accidentals (sharps and flats) of all the notes in that scales, as shown in the last column. (That's actually the reason for showing the "duplicate" scales in this table, so that this pattern will be clear.) Music written by selecting notes from one of these scales is said to be written in that *key*. For example, a musical composition written using notes selected from the C major scale is said to be written "in the key of C major". This selection of notes is not strictly adhered to, though; while the notes in a composition are generally selected from the seven in the key being used, the composer may occasionally use other notes for effect.

Since each major key can be uniquely identified by the number of accidentals, the key in which a composition may be indicated by writing the appropriate number of sharps or flats immediately after the clef sign. For example, suppose we wish to write a composition in the key of G major. From Table 3, we see that the key of G major contains only one "sharp" note, F♯. So we indicate a key of G major by writing a single ♯ sign on the F line immediately after the clef sign; this is called the *key signature*. The performer who plays the music will see that the key signature shows a single ♯ on the F line, and will know that the key is therefore G major and that all written F notes should be played as F♯.

The major scales we've just seen are just one of many such scales, each of which gives a different "feel" to the music. For example, there are several *minor scales*; music written in a minor scale has a distinctively dark, "sad" sound to it, and may remind the listener of "spooky" or "funeral" music. There is a *whole tone scale* that is often used for jazz music, and has a whole step between each note in the scale. The *pentatonic scale* is widely used in Eastern music and for many other forms of music around the world.

Table 4 shows some of these scales, and their corresponding pitch interval patterns. Remember that each scale shown represents 12 different keys, each one starting with a different note in the chromatic scale, and each one having a bit of a different feel to it.

Table 4. Several musical scales and modes, and their pitch interval patterns. (H=half step, W=whole step, 3=three half steps.)

Name	Pattern
Major scale	WWHWWWH
Natural minor scale	WHWWHWW
Harmonic minor scale	WHWWH3H
Melodic minor scale	WHWWWWH
Whole tone scale	WWWWWW
Pentatonic scale	WW3W3
Ionian mode	WHWWWHW
Phrygian mode	HWWWHWW
Lydian mode	WWWHWWH
Mixolydian mode	WWHWWHW
Aeolian mode	WHWWHWW
Locrian mode	HWWHWWW

3 Music Notation

Suppose we wish to record a musical composition so that a musician can play it. How do we write out the notes to be played? We could just list the notes to be played (B_4, D_3, etc.), but musicians would find that difficult to read. Also, there needs to be some way to show the *duration* of each note, and to indicate when the

performer should pause while playing the composition. To deal with these issues, musicians have developed a special graphical system of musical notation to record music and indicate how it should be played.

The notation begins with five horizontal lines (called a *staff*), which essentially form a plot of frequency vs. time, with increasing frequency (pitch) going up, and increasing time to the right. Each note is written either *on* one of the lines, or in the space *between* lines. A *clef sign* is written at the beginning of the staff to indicate which lines correspond to which notes.

Two clef signs are in common use. A *treble clef* is used when writing music for women's voices, or for instruments that play high notes, generally in the range above middle C. The treble clef sign is a stylized script letter G that curlicues around the line for the note G_4. Each of the notes in the C major scale (C, D, E, F, G, A, and B) is written on or between lines of the staff; the other notes are are written using ♯ and ♭ signs next to these notes. Fig. 1 shows a treble clef symbol on the far left, followed by ovals (*whole notes*) that show how each of the notes is written for one octave. (Notice that for the first two notes, the staff has been extended downward by a short *ledger line* to write middle C and $C_4♯$.)

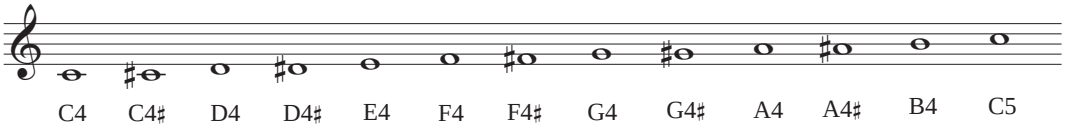

Figure 1: Treble clef showing the chromatic scale for octave 4 (plus C_5). The lowest note (far left) is middle C.

A *bass clef* is used for men's voices, or for instruments that play low notes, generally below middle C. The bass clef sign is a stylized script letter F, with two dots on either side of the line for the note F_3. Again every line or space corresponds to one of the notes in the C major scale, with ♯ and ♭ signs written for the other notes. Fig. 2 shows a bass clef symbol on the far left, along with whole notes showing each note for one octave.

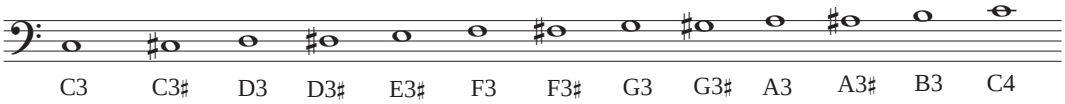

Figure 2: Bass clef showing the chromatic scale for octave 3, plus middle C (C_4) on the far right.

A few other clef signs are in use. For example, viola music is written using an *alto clef*, and there is a *tenor clef* that is common in vocal music. The main point of the different clefs is to shift the notes that are assigned to the lines of the staff in order to minimize the number of ledger lines that are needed. Music is easier to read if most of the notes lie within the five lines of the staff.

The *duration* of each note in time is indicated by various symbols, as shown in Fig. 3. A *whole note*, drawn as an oval as shown on the far left, is the longest duration. Other notes are fractions of a whole note, as shown in the figure: a *half note* has half the duration of a whole note, a *quarter note* has one-fourth the duration of a whole note, and so on. So each written note indicates a specific pitch (by its position on the staff) and a specific duration in time (by the symbol used).

Similarly, there are symbols for pauses, or *rests*. Fig. 4 shows the various symbols for rests of different durations. The longest rest (a *whole rest*) is shown on the far left. Next is a *half rest*, which has half the duration of a whole rest, and so on. The rests are always placed on the staff as shown in the figure; they are not drawn higher or lower on the staff, since there is no pitch to be indicated.

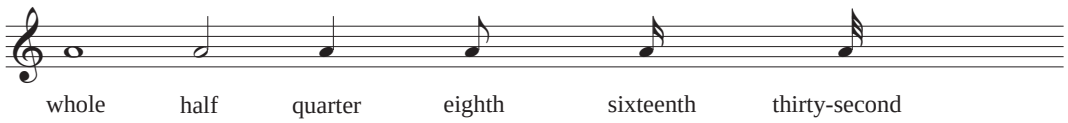

Figure 3: Note A_4, showing the symbols for different note durations. The stems may point either upward (as shown here) or downward; generally they point upward for notes near the bottom of the staff, and downward for notes near the top.

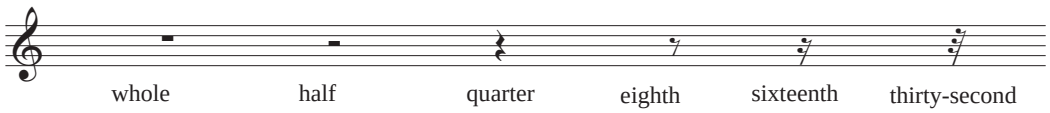

Figure 4: Symbols for different rest durations.

4 Timing

When we specified the durations of different notes, we specified them relative to the length of a whole note. But what is the duration of a whole note, in seconds? That's not necessarily specified—music may be played faster or slower, so the duration of a whole note is somewhat flexible. But if he wishes, a composer may indicate a specific rate, or *tempo*, at which the music is to be played (typically in units of quarter notes per minute).

For convenience, musical notes are grouped into *measures* of equal time; these are indicated by vertical lines dividing the staff. A *time signature* immediately follows the key signature, and indicates how the timing of the composition works. The time signature is written as something resembling a fraction of the form p/q, where p is the number of "beats" of music per measure, and q indicates which note represents one beat. Some common time signatures are shown in Table 5.

Table 5. Some time signatures.

Time Signature	Beats per measure	1 beat =
$\frac{4}{4}$ (or C)	4	quarter note
$\frac{2}{2}$ (or ₵)	2	half note
$\frac{2}{4}$	2	quarter note
$\frac{3}{4}$	3	quarter note
$\frac{6}{8}$	6	eighth note

5 An Example

Figure 5 shows a simple example of musical notation—the beginning of a popular song, *Old MacDonald Had a Farm*.

Let's break this down and see how it all works. Starting at the far left, you see the treble clef, which indicates which lines on the staff correspond to which notes. Immediately after the treble clef is the key

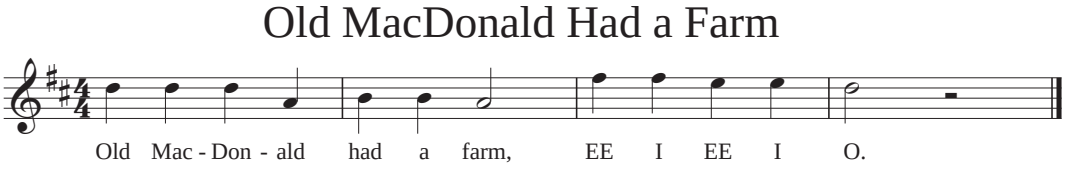

Figure 5: The first 12 notes of *Old MacDonald Had a Farm*, in D major.

signature, which is two ♯ (sharp) signs on the line for note F_5 and the space for C_5. As shown in Table 3, the key with two ♯ signs is the key of D major, and in that key the sharp notes are F♯ and C♯. In the key of D major, all written F notes are to be played as F♯, and all written C notes are to be played as C♯.

After the key signature comes the time signature, which is $\frac{4}{4}$ in this case—meaning four quarter notes (or the equivalent) per measure. After the time signature we see the notes: D_5, D_5, D_5, A_4; B_4, B_4, A_4; F_5♯, F_5♯, E_5, E_5; and D_5. The words (*lyrics*) are written below the staff.

6 Musical Instruments

Musical instruments produce musical notes by creating standing waves of some sort. In *string instruments* (violin, cello, guitar, harp, etc.), a string under tension is caused to vibrate, either by being plucked or having a bow drawn across it. The string is held fixed at both ends, and standing waves are created in the string, which produces a sound. Recall that the frequencies f_n of standing waves fixed at both ends are given by

$$f_n = n\frac{v}{2L} \qquad (n = 1, 2, 3, 4, \ldots) \tag{2}$$

where v is the wave speed and L is the distance between the ends. Only the first harmonic ($n = 1$) standing wave is played on a string instrument. Recall also that the speed of waves in a string is given by $v = \sqrt{F_T/(m/L)}$ (where F_T is the tension and m/L is the string density), so the frequency of the first harmonic will be

$$f_1 = \frac{1}{2L}\sqrt{\frac{F_T}{m/L}}. \tag{3}$$

The performer can shorten the effective length L of the string, typically by pressing the string against the neck of the instrument. Since m/L is constant, we have $f_1 \propto 1/L$, and shortening the string will increase the pitch f_1 and play a higher note. String instruments will have several strings with different thicknesses; the thicker strings have a higher mass density m/L, so they play a lower pitch. In order to tune the instrument before playing, a set of knobs allows the player to change the tension F_T in each string to make sure it plays each note at the proper frequency; a higher tension gives a higher pitch.

In *brass instruments* (e.g. trumpet, trombone, French horn, tuba), the performer sets up standing sound waves in the instrument by blowing into a mouthpiece. The player's lips vibrate or "buzz" at a frequency that produces standing waves; different notes are produced by changing the length of tubing (using valves, or a slide for the trombone), and by changing the tension in the player's lips. In some brass instruments, like the trombone, the player can play the first harmonic by buzzing the lips very loosely in the mouthpiece; higher harmonics are produced by increasing the lip tension. In other instruments, like the French horn, the first harmonic cannot be played—only higher harmonics. This makes the French horn a tricky instrument to play—only slight changes in lip tension will change the note from one harmonic from the next.

In *woodwind instruments* (e.g. clarinet, oboe, bassoon, flute, recorder), as in brass instruments, the player sets up standing sound waves in the instrument. In this case, the vibrations are often produced with a reed, and

the performer changes notes by opening or closing combinations of holes along the side of the instrument, using the fingers or a complex system of keys. Woodwind instruments generally play the first few harmonic standing waves; which ones can be played depend on the shape of the bore of the instrument.

Percussion instruments are instruments like drums, which produce a sound when a membrane or other surface is struck and allowed to vibrate, creating standing waves in the membrane. The timpani is a drum in which the tension in the membrane can be changed to produce a few different notes.

Some musical instruments are *transposing* instruments; for these instruments, the written notes are not the same as the notes that are actually played. For example, music for the French horn is written seven half steps higher than it is actually played. So when a French horn player plays a written middle C (C_4), the note that actually comes out of the instrument will be seven half steps lower, F_3; such a horn is said to be "pitched in F", and is called an *F horn*. There is a lighter French horn favored by some players that is better for playing high notes; it plays a B♭ for a written C, and is called a *B♭ horn*. The horn most commonly seen in orchestras, with its very complex-looking system of tubing, is a *double horn*. The double horn contains tubing for *both* an F horn and a B♭ horn, and allows the player to switch between the two sides using a thumb valve. The player will play lower notes on the F side of the horn, then use the thumb valve to switch to the B♭ side for high notes, since they're easier to play on that side. Today there's an even more complex *triple horn*, which includes a third *descant horn* side for playing very high notes.

Transposition is partly for historical reasons, and partly to allow performers to play similar instruments more easily. For example, a trumpet player can play a French horn or tuba without having to learn a different fingering for each instrument. However, if a performer wishes to play music written for an instrument other than the one he is playing (a horn player playing music written for trombone, for example), he may need to mentally transpose the music while playing in order to play in the same key as the rest of the orchestra.

As mentioned earlier, music is a very large subject, and here we've only barely touched on the very basics of music theory and musical notation. There's much more to this subject: chords, harmony, timbre, intervals, non-Western music, etc.—and there's much more to musical notation than the bare outlines we've seen here. The interested reader is referred to books on music theory for more information.