Physics Recreations:
Galileo's Law

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1 Introduction

Galileo Galilei (1564–1642) was an Italian physicist to did some of the early work in classical mechanics. One of his major contributions was in the area of falling bodies. Galileo recognized that bodies falling due to the Earth's gravity will fall with a constant acceleration, and that he could study that acceleration by, in effect, slowing it down through the use of an inclined plane.

Galileo constructed an inclined plane tilted at a slight angle ($4°$), with a groove in the center. He then rolled a solid brass ball in the groove down incline and studied how the ball moved as a function of time. He could do this by placing small bumps in the groove in which the ball rolled; whenever the ball hit a bump, it made a noise. By studying the timing of the noises and comparing that to the distance between the bumps, he could make some quantitative studies of the ball accelerating down the incline.

Unfortunately, Galileo did not have access to accurate timepieces—using his own pulse was about the best method available. If he made the bumps equally spaced apart along the incline, he could tell that the noises of the ball hitting the bumps got closer together as the ball rolled down the incline, but had no way to measure the times accurately.

Then Galileo hit upon an idea: instead of spacing the bumps equally far apart, he would adjust their spacing until he could hear that the *time* between the ball hitting the bumps was the same. As a skilled player of the Renaissance lute, Galileo had a well-developed sense of musical rhythm, and was able to judge fairly accurately when the *click-click-click-click* of the ball rolling down the incline and hitting the bumps on the incline were separated by equal time intervals. Once he was satisfied that the sounds of the balls hittings the bumps were all equally separated in time, he could accurately measure the distances between the bumps.

He discovered that the distance from the top of the incline to the second bump was 4 times the distance to the first bump; the distance to the third bump was 9 times the distance to the first bump; the distance to the fourth bump was 16 times the distance to the first bump, and so on. This allowed him to deduce what is now called *Galileo's Law*: the total distance x covered in time t is proportional to the square of the time:

$$\boxed{x \propto t^2} \tag{1}$$

2 Modern Treatment

Developments in the theory of classical mechanics since Galileo's time allow us to investigate his experiment in more detail. For one thing, we now know that the proportionality constant in Eq. (1) is $a/2$, where a is the acceleration of the ball down the incline; Galileo's Law then becomes

$$x = \frac{1}{2}\, at^2. \tag{2}$$

Furthermore, we now know that the acceleration a of a solid ball rolling down an inclined plane is given by

$$a = \frac{g \sin\theta}{1 + \frac{I_{\rm cm}}{MR^2}}, \tag{3}$$

where g is the acceleration due to gravity (9.8 m/s^2), θ is the inclination of the inclined plane, $I_{\rm cm}$ is the moment of inertia of the ball about its center of mass, M is the mass of the ball, and R is the radius of the ball. For a solid spherical ball, we know

$$I_{\rm cm} = \frac{2}{5}\, MR^2, \tag{4}$$

so $I_{\rm cm}/MR^2 = 2/5$; the acceleration of a solid ball down an inclined plane is therefore

$$a = \frac{5}{7}\, g \sin\theta. \tag{5}$$

Galileo's Law for a solid ball rolling down an incline then becomes

$$\begin{aligned} x &= \frac{1}{2}\, at^2 && (6) \\ &= \frac{1}{2} \left(\frac{5}{7}\, g \sin\theta \right) t^2 && (7) \\ &= \frac{5}{14}\, (g \sin\theta)\, t^2. && (8) \end{aligned}$$

Using $g = 9.8$ m/s^2 and $\theta = 4^\circ$ for Galileo's incline, we get

$$x = 0.244\, t^2, \tag{9}$$

where x is in meters and t is in seconds.