

Physics Recreations: Helmert's Equation

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We use a value of the acceleration due to gravity (g) of 9.8 m/s^2 . But what if we want to use a more exact value?

You might be tempted to use the value given in some reference books: $g = 9.80665 \text{ m/s}^2$, but that would actually be wrong. This value is just a standard value used for the definitions of some units (for example, in the conversion between pounds-force and newtons). You should never use this value in a physics formula that contains g as the acceleration due to gravity—it's only used when doing certain unit conversions.

The acceleration due to gravity g at the surface of the Earth varies over the surface of the Earth for a number of reasons:

1. As you get closer to the equator, the Earth's rotation rate gets larger, resulting in a greater centrifugal force that counteracts gravity. This has the effect of reducing g closer to the equator.
2. Also, the Earth has an equatorial bulge due to its rotation, so that you're farther from the center of the Earth near the equator. This also has the effect of reducing g closer to the equator.
3. There is also an elevation effect: the higher you are in elevation, the smaller g is.

These effects can be approximately accounted for using an equation called *Helmert's equation*. According to Helmert's equation, the acceleration due to gravity is given by

$$g = 9.80616 - 0.025928 \cos 2\phi + (6.9 \times 10^{-5}) \cos^2 2\phi - (3.086 \times 10^{-6})H \text{ m/s}^2,$$

where ϕ is the latitude and H is the elevation (in meters) above sea level. For example, for Largo, Maryland, the latitude ϕ is $38^\circ 898$ and the elevation H is about 174 feet (53.0 meters). Substituting these values into Helmert's equation, we find g at Largo is about 9.8005 m/s^2 . In other cities around the world, the value ranges from 9.779 m/s^2 (Mexico City) to 9.819 m/s^2 (Helsinki). For most problems we just use an average value of 9.8 m/s^2 . (You should *never* round this to 10 m/s^2 unless you're doing a very rough order-of-magnitude estimation.)