Physics Recreations: Creating Your Own System of Units

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As we've seen, there are many systems of units in current use (SI, Gaussian, electromagnetic, etc.). If you like, you can create your own system of units as an interesting exercise.

SI units defines seven base units (m, kg, s, A, K, mol, cd), but there is nothing particularly significant about the number 7 — you could define a system of units with more or fewer base units than this. CGS units, for example, have no base electrical unit, and the "natural" system of units used by particle physicists has only *one* base unit (an energy unit in that case). The tradeoff is that if you define fewer base units, you will have to define the values of some of the physical constants.

To see how this is done, let's create, for example, a system of units based entirely on one base unit — using the *cubit* (which we'll abbreviate *cub*), an ancient unit equal to 18 inches, as the base unit of length.

How would we measure time in this system? We can define the unit of time to be the amount of time it takes light to travel one cubit in vacuum. Then the speed of light in vacuum $c = 1$ (c will be *dimensionless*, i.e. have no units), and all other velocities in this system will also be dimensionless. Since $v = x/t$, this means that time in this system will be measured in units of cubits. In terms of SI units, 1 "cubit" of time would be 18 in/c = 1.525 nanoseconds.

How would we measure mass in this system? Again, we'll have to define the value of a physical constant. A convenient choice would be to define Planck's constant $\hbar = 1$ (so $\hbar$ will also dimensionless). In SI units, Planck's constant has units of kg m^2 s^{-1}, or mass $\times$ length^2 / time. In our cubit-based system, this would mean mass $\times$ cub^2 / cub must be dimensionless, and so mass must be measured in units of cub^{-1}. In terms of SI units, one cub^{-1} of mass would be equivalent to $\hbar/(c \times 18\text{in})$, or 7.694×10^{-43} kg.

For temperature, we can define Boltzmann's constant $k_B = 1$ (again, dimensionless). Boltzmann's constant has SI units of kg m^2 s^{-2} K^{-1}, or mass $\times$ length^2 / (time^2 $\times$ temperature). In our cubit-based system, this would mean (cub^{-1}) cub^2 / (cub^2 $\times$ temperature) must be dimensionless, so temperature would have units of cub^{-1}.

You could continue in this way, defining cubit-based units for measuring electric charge, electric current, magnetic field, etc. Or you can define another system — say using the amount of time it takes to blink your eyes (the *blink*) as the base unit of time, and define everything based on that. Or you could come up with your own definitions of a base time and mass unit, and define a system based on that.

In practice, units like our cubit-based system or the "natural" units used by particle physicists are mostly useful in purely theoretical work: they do simplify many equations, since a number of physical constants are just equal to 1. But if we want to have a *practical* way to measure time, it's more convenient to define a unit of time, like the second, that can be reproduced by experiment.