

FORMULÆ

Physics 1030 Final Exam

$$\rho = \frac{M}{V}$$

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt} \quad \Rightarrow \quad x(t) = \int v(t) dt$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad \Rightarrow \quad v(t) = \int a(t) dt$$

$$x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

$$v(t) = at + v_0$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} (+A_z \mathbf{k})$$

$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\begin{cases} A_x &= A \cos \theta \\ A_y &= A \sin \theta \end{cases}$$

$$\begin{cases} |\mathbf{A}| = A &= \sqrt{A_x^2 + A_y^2} \\ \tan \theta &= \frac{A_y}{A_x} \end{cases}$$

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \quad \Rightarrow \quad \mathbf{r}(t) = \int \mathbf{v}(t) dt$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \quad \Rightarrow \quad \mathbf{v}(t) = \int \mathbf{a}(t) dt$$

Circle:

$$C = 2\pi r = \pi d$$

$$A = \pi r^2 = \frac{\pi}{4}d^2$$

Sphere:

$$A = 4\pi r^2 = \pi d^2$$

$$V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}d^3$$

Constants:

$$g = 9.80 \text{ m/s}^2$$

$$\rho_{\text{water}} = 1.00 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$$

$$1 \text{ atm} = 101,325 \text{ Pa}$$

$$H = 8 \text{ km (atmosphere scale height)}$$

$$\mathbf{r}(t) = \frac{1}{2}\mathbf{a}t^2 + \mathbf{v}_0t + \mathbf{r}_0$$

$$\mathbf{v}(t) = \mathbf{a}t + \mathbf{v}_0$$

$$v^2 = v_0^2 + 2\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_0)$$

Summary of formulæ for projectile motion.

Quantity	Formula
$x(t)$	$x = (v_0 \cos \theta)t$
$y(t)$	$y = -\frac{1}{2}gt^2 + (v_0 \sin \theta)t$
$y(x)$	$y(x) = \left(-\frac{g}{2v_0^2 \cos^2 \theta}\right)x^2 + (\tan \theta)x$
Time in flight	$t_f = \frac{2}{g}v_0 \sin \theta$
Range at angle θ	$R = \frac{v_0^2}{g} \sin 2\theta$
Max. range (at $\theta = 45^\circ$)	$R_{\max} = \frac{v_0^2}{g}$
Angle needed to hit target at range R for fixed v_0	$\theta = \frac{1}{2} \sin^{-1} \left(\frac{gR}{v_0^2} \right)$
Speed needed to hit target at range R for fixed θ	$v_0 = \sqrt{\frac{gR}{\sin 2\theta}}$
Max. altitude	$h = \frac{v_0^2 \sin^2 \theta}{2g}$
Speed needed to hit target at (x_t, y_t) for fixed θ	$v_0 = \sqrt{\frac{gx_t}{2\left(\tan \theta - \frac{y_t}{x_t}\right) \cos^2 \theta}}$
Angle needed to hit target at (x_t, y_t) for fixed v_0	$x_t \sin 2\theta - 2y_t \cos^2 \theta = \frac{gx_t^2}{v_0^2}$

$$W = mg$$

$$\sum \mathbf{F} = m\mathbf{a}$$

$$a = g \sin \theta$$

$$f_s \leq \mu_s n$$

$$f_k = \mu_k n$$

$$\mu_s = \tan \theta_s$$

$$\mu_k = \tan \theta_k$$

$$v_\infty = \sqrt{\frac{2mg}{C_D \rho A}}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

Formulae for computing work.

Formula	$\mathbf{F} \parallel \mathbf{r}$?	Constant $\mathbf{F}$?
$W = Fx$	✓	✓
$W = \mathbf{F} \cdot \mathbf{r}$		✓
$W = \int F dx$	✓	
$W = \int \mathbf{F} \cdot d\mathbf{r}$		

Formulae for potential energy.

Force	Formula
Gravity	$U = -\frac{Gm_1m_2}{r}$
Gravity (near Earth surface)	$U = mgh$
Electric	$U = \frac{q_1q_2}{4\pi\epsilon_0 r}$
Elastic (spring)	$U = \frac{1}{2}kx^2$

$$\text{Period } T = 2\pi/\omega$$

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}I\omega^2$$

$$F \propto r^n \Rightarrow \langle K \rangle = \frac{n+1}{2} \langle U \rangle$$

$$\mathcal{P} = \frac{dE}{dt}$$

$$I = \int r^2 dm$$

$$\lambda(x) = \frac{dm}{dx}$$

$$I = I_{\text{cm}} + Mh^2$$

$$\beta \equiv \frac{I_{\text{cm}}}{MR^2}$$

$$v = \sqrt{\frac{2gh}{\beta + 1}}$$

$$a = \frac{g \sin \theta}{\beta + 1}$$

$$s = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}=I\boldsymbol{\alpha}$$

$$M=E-e\sin E$$

$$L=I\omega$$

$$\frac{dP}{dh}=\rho g$$

$$\begin{aligned}\theta(t) &= \tfrac{1}{2}\alpha t^2 + \omega_0 t + \theta_0 \\ \omega(t) &= \alpha t + \omega_0 \\ \omega^2 &= \omega_0^2 + 2\alpha(\theta - \theta_0)\end{aligned}$$

$$P=P_0+\rho gh$$

$$P=P_0e^{-y/H}$$

$$T=2\pi\sqrt{\frac{L}{g}}$$

$$\mathbf{p}=m\mathbf{v}$$

$$\mathbf{F}=\frac{d\mathbf{p}}{dt}$$

$$F_{\mathrm{ave}}=\frac{I}{\Delta t}$$

$$I=\int F\,dt\,=\Delta p$$

$$\epsilon=\frac{p_f}{p_i}=\sqrt{\frac{h_f}{h_i}}$$

$$\Delta v=v_p\ln\frac{m}{m_e}$$

$$x_{\mathrm{cm}}=\frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$y_{\mathrm{cm}}=\frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$x_{\mathrm{cm}}=\frac{\int x\,\lambda(x)\,dx}{\int \lambda(x)\,dx}$$

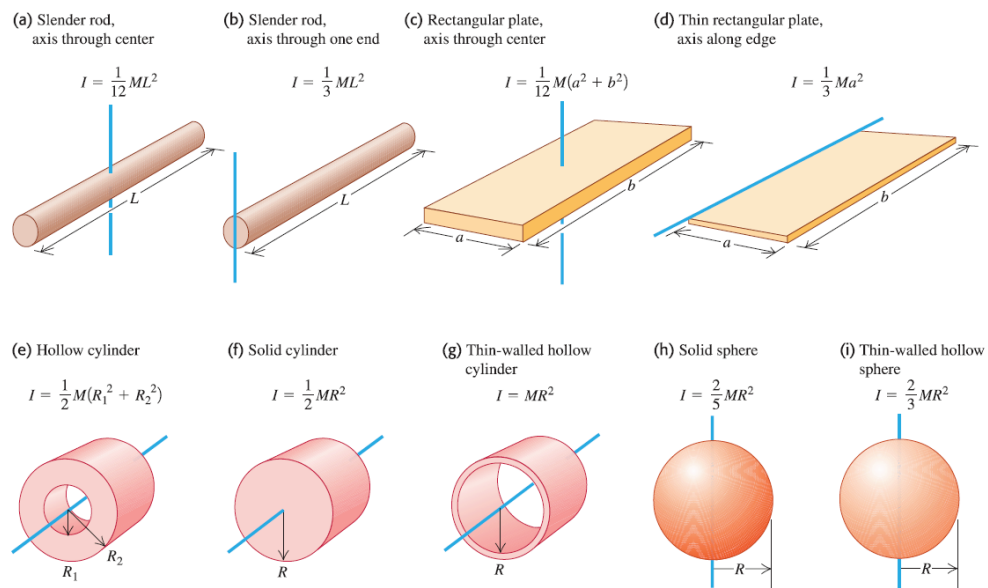


Figure 1: Table of moments of inertia.