PHY 157
The Simple Pendulum
(Experiment 3)

Name: ______________________________

1 Introduction

A simple pendulum consists of a small mass m (called the pendulum *bob*) attached to the end of a string of length L. In the simple theory of the pendulum, the mass of the string is ignored. Length L is the distance from the point of support to the *center of mass* of the pendulum bob. If the angular coordinate θ is used to describe the *displacement* of the pendulum, then the *amplitude* of the pendulum is the maximum displacement, θ_0. Once complete repetition of the motion (one round trip) is called a *cycle* of the motion and the time for the pendulum to complete a cycle is called its *period*.

2 Apparatus

- Pendulum apparatus
- Protractor
- Pasco photogate timer
- meter stick

3 Procedure

Switch the photogate timer to PENDULUM MODE, which sets the timer to record the time for one cycle of the motion. The first pass of the bob through the gate starts the timer, the return pass moving in the opposite direction is ignored, and the next pass going in the original direction stops the timer. Set the RESOLUTION SWITCH on the bottom of the timer to 1 ms. To properly measure the period, you must adjust the height of the photogate head so that the center of the bob (i.e. its diameter) passes through the exact center of the photocell beam. *The pendulum must be carefully released to ensure that the bob does not hit the timer!* Also be sure to record all significant figures of the measured data.

1. **Setup.** Adjust one of the metal pendula to a length of about 80 cm and adjust the height of the suspension point so that the bob passes through the center of the photogate timer.

2. **Period vs. Amplitude.** To test rigorously the relationship between period and amplitude, use the following table:

θ_0 (deg)	Period (sec)

   (a) Release the pendulum from an amplitude of 5 to 8 degrees and determine its period. Repeat for approximate amplitudes of 15, 30, and 45 degrees.

   (b) Examine the results. Do your measurements indicate a relationship between amplitude and period?

   If so, does the period increase or decrease with amplitude?

3. **Period vs. Mass** Select two pendula of *unequal mass* and carefully adjust bot the the same length (about 80 cm).

   (a) Release the two pendula simultaneously from the same amplitude (about 10 degrees) and observed the for several cycles.

   (b) From your observations, does the period of the pendulum seem to depend upon the *mass* of its bob?

   If so, which pendulum has the longer period?

4. **Period vs. Length.** To more closely examine the relationship between period and pendulum length, use the following table:

Pendulum length (cm)	Period (sec)

   (a) Adjust the length of the pendulum to approximately 80 cm and then carefully measure its length. Recall that its length is from the point of support to the center of the bob. Record its length to the appropriate number of significant figures.

   (b) Release the pendulum from an amplitude of about 5 to 8 degrees and measure its period.

   (c) Then lower the pendulum and adjust its length to approximately 75 cm. Verifiy that the bob again passes through the center of the photogate. Measure and record its length. release the pendulum from and amplitude of about 5 to 8 degrees and determine its period.

   (d) Repeat the measurements for approximate lengths of 60, 40, and 20 cm.

4 Analysis

Some of the observations and data that you obtained above will be subjected to further analysis, to be done in this section.

1. Of the three variables associated with the simple pendulum (amplitude, mass, and length), which variable has the greatest influence on the period?

   Which variables, if any, seem to have no influence on the period?

2. If you discovered that the period does vary with amplitude, determine the percent change in period from the smallest amplitude to the largest amplitude.

3. **Analysis: Period vs. Length data.** The period vs. length data can be further analyzed to determine the type of relationship between the variables.

   (a) *Graph the data.* Using a graphig calculator, computer, or graph paper, make a graph of your period data (in seconds) as a function of length (in cm). Draw a sketch of the resulting graph. Is the relationship appear to be linear? (It should not be.)

If the data does not show a linear relationship, suppose we assume that the relationship between the period T of a pendulum and its length L can be expressed by a *power law* in the form

$$T = AL^n, \tag{1}$$

where A is a constant and n is an unknown power of L. The value of n can be either positive or negative, but for most simple situtations in physics and engineering, we expect it be be a small integer or simple rational number (e.g. 1, 2, 3, ½, ⅓, ¼, etc.).

There is an easy way to verify that a set of measured data obeys a power law. A power law can be transformed into a linear relationship by taking the logarithm (base 10) of both sides of the equation. The result for the power law expressed in equation (1) is

$$\log T = n \log L + \log A, \tag{2}$$

which is a linear function between ($\log T$) and ($\log L$) with slope n and y-intercept $\log A$. Thus the measured data obeys a power law if you can show that the graph of ($\log T$) vs. ($\log L$) is a straight line.

(b) *Graph the transformed data.* To determine if the period vs. length data does indeed obey a power law, graph ($\log T$) vs. ($\log L$) on a graphing calculator, computer, or graph paper. Does this transformed data graph as a straight line? (It should.) Draw a sketch of the resulting graph.

(c) *Linear regression.* Do a linear regression analysis on ($\log T$) vs. ($\log L$) and report the values of the slope and y-intercept.

(d) *Value of the power of n.* Look at the value of n obtained in your regression analysis and determine and record the *most likely value* for the theoretical value of n, keeping in mind that the experimental data always has some errors and is not exact.

A theoretical analysis of the simple pendulum reveals that its period is a function of the square root of its length:

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \left(\frac{L}{g}\right)^{1/2}. \tag{3}$$

Thus, in theory, n equals ½. How well does your "most likely value" agree with theory?