PHYSICS 1030

FINAL EXAM PRACTICE PROBLEMS

1. Find the period of a simple pendulum of length 3.3 meters.
2. If you drop a stone (from rest) from the top of a building of height 220 meters, then how long does it take the stone to reach the ground?
3. In the previous problem, what is the stone's impact velocity?
4. If you drop a stone (from rest) from the top of a building and it takes 5.8 seconds to reach the ground, then how high is the building?
5. State Newton's second law of motion.

For problems 6–10, take vectors $\mathbf{A} = 3\mathbf{i} + 7\mathbf{j}$, and $\mathbf{B} = 5\mathbf{i} - 4\mathbf{j}$.

6. Find $\mathbf{A} \cdot \mathbf{B}$.
7. Find $\mathbf{A} \times \mathbf{B}$.
8. Convert $\mathbf{A}$ from cartesian form to polar form.
9. Find $|\mathbf{B}|$
10. Find the angle between $\mathbf{A}$ and $\mathbf{B}$.

11. If a projectile is fired with a muzzle velocity of 50.00 m/s at an angle of 60° from the horizontal, then what is its range?
12. In the previous problem, what is the projectile's maximum altitude?
13. What is the centripetal acceleraton of a body moving in a circle of radius 7.0 m at a speed of 20 m/s?
14. A block is placed on an inclined plane. The plane must be tilted at an angle of 27° before the block begins to slide. What is the coefficient of static friction?
15. What is the work done against gravity in lifting a box of mass 7 kg from the ground to a height of 4 meters above the ground?
16. A mass of 8 kg is located on an x-axis at $x = 2.0$ cm, and a mass of 10 kg is at $x = 9$ cm. What is the x coordinate of the center of mass?
17. What is the kinetic energy of a golf ball of mass 45 g and having a speed of 55 m/s?
18. What is the moment of inertia of a solid cylinder having a mass of 25 kg and a radius of 1.8 meters?
19. If the cylinder in the previous problem is rotated at 20.0 rad/sec, then what is its rotational kinetic energy?
20. In general, is kinetic energy a conserved quantity?

21. What is the name of the curve followed by a projectile near the Earth's surface?
22. What is the name of the curve followed by a planet in orbit around the Sun?
23. Name the four fundamental fources of Nature.
24. What is Einstein's theory of gravity called?
25. Name the three conserved quantities in classical mechanics.

Answers.

1. 3.646 sec   2. 6.70 sec   3. 65.67 m/s   4. 164.8 m   5. $F = ma$ 6. -13 7. $-47\mathbf{k}$
8. $7.616\angle 66.80°$ 9. 6.403   10. 105.46°   11. 220.92 m   12. 95.66 m   13. 57.14 m/s^2
14. 0.5095   15. 274.4 J   16. $x = 5.889$ cm   17. 68.06 J   18. 40.5 kg m^2 19. 8100 J   20. No
21. parabola   22. ellipse   23. gravity; electromagnetism; strong nuclear; weak nuclear
24. general theory of relativity   25. energy, linear momentum, angular momentum

FORMULÆ
Physics 1030 Final Exam

$$\rho = \frac{M}{V}$$

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt} \qquad \Rightarrow \qquad x(t) = \int v(t)\,dt$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \qquad \Rightarrow \qquad v(t) = \int a(t)\,dt$$

$$x(t) = \tfrac{1}{2}at^2 + v_0 t + x_0$$
$$v(t) = at + v_0$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j}\ (+A_z\,\mathbf{k})$$
$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\begin{cases} A_x &= A\cos\theta \\ A_y &= A\sin\theta \end{cases}$$

$$\begin{cases} |\mathbf{A}| = A &= \sqrt{A_x^2 + A_y^2} \\ \tan\theta &= \frac{A_y}{A_x} \end{cases}$$

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \qquad \Rightarrow \qquad \mathbf{r}(t) = \int \mathbf{v}(t)\,dt$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \qquad \Rightarrow \qquad \mathbf{v}(t) = \int \mathbf{a}(t)\,dt$$

Circle:

$$C = 2\pi r = \pi d$$
$$A = \pi r^2 = \tfrac{\pi}{4}d^2$$

Sphere:

$$A = 4\pi r^2 = \pi d^2$$
$$V = \tfrac{4}{3}\pi r^3 = \tfrac{\pi}{6}d^3$$

Constants:

$$g = 9.80\ \text{m/s}^2$$
$$\rho_{\text{water}} = 1.00\ \text{g/cm}^3 = 1000\ \text{kg/m}^3$$
$$1\ \text{atm} = 101{,}325\ \text{Pa}$$
$$H = 8\ \text{km (atmosphere scale height)}$$

$$\mathbf{r}(t) = \tfrac{1}{2}\mathbf{a}t^2 + \mathbf{v}_0 t + \mathbf{r}_0$$
$$\mathbf{v}(t) = \mathbf{a}t + \mathbf{v}_0$$
$$v^2 = v_0^2 + 2\mathbf{a}\cdot(\mathbf{r} - \mathbf{r}_0)$$

Summary of formulæ for projectile motion.

Quantity	Formula
$x(t)$	$x = (v_0 \cos\theta)t$
$y(t)$	$y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t$
$y(x)$	$y(x) = \left(-\frac{g}{2v_0^2 \cos^2\theta}\right)x^2 + (\tan\theta)x$
Time in flight	$t_f = \frac{2}{g} v_0 \sin\theta$
Range at angle θ	$R = \frac{v_0^2}{g} \sin 2\theta$
Max. range (at $\theta = 45°$)	$R_{\text{max}} = \frac{v_0^2}{g}$
Angle needed to hit target at range R for fixed v_0	$\theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v_0^2}\right)$
Speed needed to hit target at range R for fixed θ	$v_0 = \sqrt{\frac{gR}{\sin 2\theta}}$
Max. altitude	$h = \frac{v_0^2 \sin^2\theta}{2g}$
Speed needed to hit target at (x_t, y_t) for fixed θ	$v_0 = \sqrt{\frac{g x_t}{2\left(\tan\theta - \frac{y_t}{x_t}\right)\cos^2\theta}}$
Angle needed to hit target at (x_t, y_t) for fixed v_0	$x_t \sin 2\theta - 2y_t \cos^2\theta = \frac{g x_t^2}{v_0^2}$

$$W = mg$$

$$\sum \mathbf{F} = m\mathbf{a}$$

$$a = g \sin\theta$$

$$f_s \le \mu_s n$$
$$f_k = \mu_k n$$

$$\mu_s = \tan\theta_s$$
$$\mu_k = \tan\theta_k$$

$$v_\infty = \sqrt{\frac{2mg}{C_D \rho A}}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

Formulæ for computing work.

Formula	$\mathbf{F} \parallel \mathbf{r}$?	Constant $\mathbf{F}$?
$W = Fx$	✓	✓
$W = \mathbf{F} \cdot \mathbf{r}$		✓
$W = \int F\,dx$	✓	
$W = \int \mathbf{F} \cdot d\mathbf{r}$		

Formulæ for potential energy.

Force	Formula
Gravity	$U = -\frac{Gm_1 m_2}{r}$
Gravity (near Earth surface)	$U = mgh$
Electric	$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$
Elastic (spring)	$U = \frac{1}{2}kx^2$

Period $T = 2\pi/\omega$

$$K = \tfrac{1}{2}mv^2$$

$$K = \tfrac{1}{2}I\omega^2$$

$$F \propto r^n \quad \Rightarrow \quad \langle K \rangle = \frac{n+1}{2}\langle U \rangle$$

$$\mathcal{P} = \frac{dE}{dt}$$

$$I = \int r^2\,dm$$

$$\lambda(x) = \frac{dm}{dx}$$

$$I = I_{\text{cm}} + Mh^2$$

$$\beta \equiv \frac{I_{\text{cm}}}{MR^2}$$

$$v = \sqrt{\frac{2gh}{\beta + 1}}$$

$$a = \frac{g\sin\theta}{\beta + 1}$$

$$s = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha}$$

$$L = I\omega$$

$$\theta(t) = \tfrac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$$
$$\omega(t) = \alpha t + \omega_0$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$F_{\text{ave}} = \frac{I}{\Delta t}$$

$$I = \int F\,dt = \Delta p$$

$$\epsilon = \frac{p_f}{p_i} = \sqrt{\frac{h_f}{h_i}}$$

$$\Delta v = v_p \ln\frac{m}{m_e}$$

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$
$$y_{\text{cm}} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$x_{\text{cm}} = \frac{\int x\,\lambda(x)\,dx}{\int \lambda(x)\,dx}$$

$$M = E - e\sin E$$

$$\frac{dP}{dh} = \rho g$$

$$P = P_0 + \rho g h$$

$$P = P_0 e^{-y/H}$$

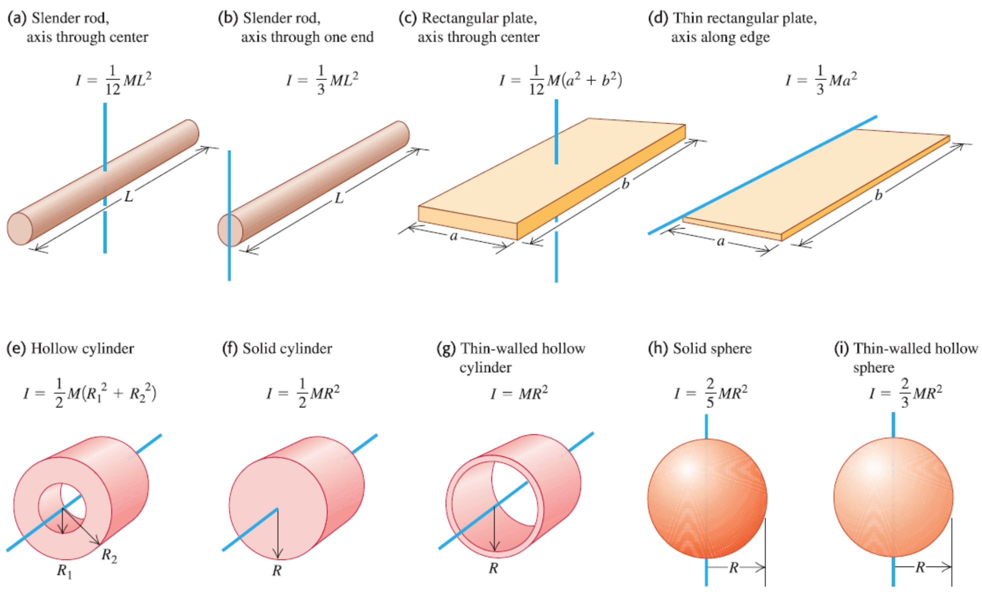

Figure 1: Table of moments of inertia.