FORMULÆ
Physics 1030 Final Exam

$$\rho = \frac{M}{V}$$

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt} \qquad \Rightarrow \qquad x(t) = \int v(t)\, dt$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \qquad \Rightarrow \qquad v(t) = \int a(t)\, dt$$

$$x(t) = \tfrac{1}{2}at^2 + v_0 t + x_0$$
$$v(t) = at + v_0$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j}\ (+A_z\,\mathbf{k})$$
$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\begin{cases} A_x &= A\cos\theta \\ A_y &= A\sin\theta \end{cases}$$

$$\begin{cases} |\mathbf{A}| = A &= \sqrt{A_x^2 + A_y^2} \\ \tan\theta &= \frac{A_y}{A_x} \end{cases}$$

$$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$
$$= (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \qquad \Rightarrow \qquad \mathbf{r}(t) = \int \mathbf{v}(t)\, dt$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \qquad \Rightarrow \qquad \mathbf{v}(t) = \int \mathbf{a}(t)\, dt$$

Circle:

$$C = 2\pi r = \pi d$$
$$A = \pi r^2 = \tfrac{\pi}{4}d^2$$

Sphere:

$$A = 4\pi r^2 = \pi d^2$$
$$V = \tfrac{4}{3}\pi r^3 = \tfrac{\pi}{6}d^3$$

Constants:

$$g = 9.80\ \text{m/s}^2$$
$$\rho_{\text{water}} = 1.00\ \text{g/cm}^3 = 1000\ \text{kg/m}^3$$
$$1\ \text{atm} = 101{,}325\ \text{Pa}$$
$$H = 8\ \text{km (atmosphere scale height)}$$

$$\mathbf{r}(t) = \tfrac{1}{2}\mathbf{a}t^2 + \mathbf{v}_0 t + \mathbf{r}_0$$
$$\mathbf{v}(t) = \mathbf{a}t + \mathbf{v}_0$$
$$v^2 = v_0^2 + 2\mathbf{a}\cdot(\mathbf{r} - \mathbf{r}_0)$$

Summary of formulæ for projectile motion.

Quantity	Formula
$x(t)$	$x = (v_0 \cos\theta)t$
$y(t)$	$y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)t$
$y(x)$	$y(x) = \left(-\frac{g}{2v_0^2\cos^2\theta}\right)x^2 + (\tan\theta)x$
Time in flight	$t_f = \frac{2}{g}v_0 \sin\theta$
Range at angle θ	$R = \frac{v_0^2}{g}\sin 2\theta$
Max. range (at $\theta = 45°$)	$R_{\max} = \frac{v_0^2}{g}$
Angle needed to hit target at range R for fixed v_0	$\theta = \frac{1}{2}\sin^{-1}\left(\frac{gR}{v_0^2}\right)$
Speed needed to hit target at range R for fixed θ	$v_0 = \sqrt{\frac{gR}{\sin 2\theta}}$
Max. altitude	$h = \frac{v_0^2 \sin^2\theta}{2g}$
Speed needed to hit target at (x_t, y_t) for fixed θ	$v_0 = \sqrt{\frac{g x_t}{2\left(\tan\theta - \frac{y_t}{x_t}\right)\cos^2\theta}}$
Angle needed to hit target at (x_t, y_t) for fixed v_0	$x_t \sin 2\theta - 2y_t \cos^2\theta = \frac{g x_t^2}{v_0^2}$

$$W = mg$$

$$\sum \mathbf{F} = m\mathbf{a}$$

$$a = g \sin\theta$$

$$f_s \le \mu_s n$$
$$f_k = \mu_k n$$

$$\mu_s = \tan\theta_s$$
$$\mu_k = \tan\theta_k$$

$$v_\infty = \sqrt{\frac{2mg}{C_D \rho A}}$$

$$a_c = \frac{v^2}{r}$$

$$F_c = \frac{mv^2}{r}$$

Formulæ for computing work.

Formula	$\mathbf{F} \parallel \mathbf{r}$?	Constant $\mathbf{F}$?
$W = Fx$	✓	✓
$W = \mathbf{F}\cdot\mathbf{r}$		✓
$W = \int F\,dx$	✓	
$W = \int \mathbf{F}\cdot d\mathbf{r}$		

Formulæ for potential energy.

Force	Formula
Gravity	$U = -\frac{Gm_1m_2}{r}$
Gravity (near Earth surface)	$U = mgh$
Electric	$U = \frac{q_1q_2}{4\pi\varepsilon_0 r}$
Elastic (spring)	$U = \frac{1}{2}kx^2$

Period $T = 2\pi/\omega$

$$K = \tfrac{1}{2}mv^2$$

$$K = \tfrac{1}{2}I\omega^2$$

$$F \propto r^n \quad \Rightarrow \quad \langle K \rangle = \frac{n+1}{2}\langle U \rangle$$

$$\mathcal{P} = \frac{dE}{dt}$$

$$I = \int r^2\,dm$$

$$\lambda(x) = \frac{dm}{dx}$$

$$I = I_{\text{cm}} + Mh^2$$

$$\beta \equiv \frac{I_{\text{cm}}}{MR^2}$$

$$v = \sqrt{\frac{2gh}{\beta+1}}$$

$$a = \frac{g\sin\theta}{\beta+1}$$

$$s = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = I\boldsymbol{\alpha}$$

$$L = I\omega$$

$$\theta(t) = \tfrac{1}{2}\alpha t^2 + \omega_0 t + \theta_0$$
$$\omega(t) = \alpha t + \omega_0$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\mathbf{p} = m\mathbf{v}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$F_{\text{ave}} = \frac{I}{\Delta t}$$

$$I = \int F\,dt = \Delta p$$

$$\epsilon = \frac{p_f}{p_i} = \sqrt{\frac{h_f}{h_i}}$$

$$\Delta v = v_p \ln\frac{m}{m_e}$$

$$x_{\text{cm}} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$
$$y_{\text{cm}} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

$$x_{\text{cm}} = \frac{\int x\,\lambda(x)\,dx}{\int \lambda(x)\,dx}$$

$$M = E - e\sin E$$

$$\frac{dP}{dh} = \rho g$$

$$P = P_0 + \rho g h$$

$$P = P_0 e^{-y/H}$$

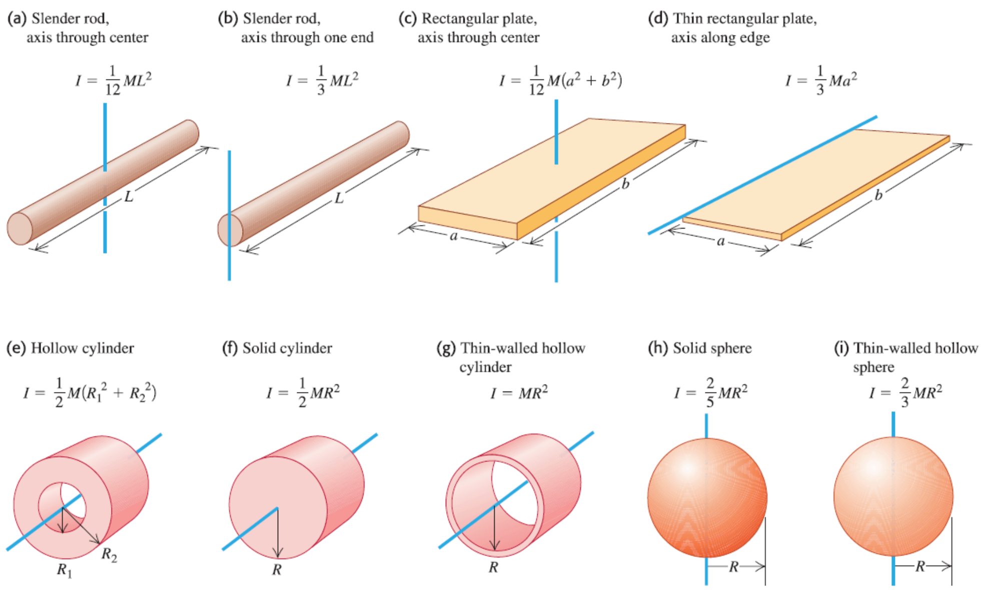

Figure 1: Table of moments of inertia.