PHYSICS 1030
Homework #8

(Due Dec. 8, 2016)

Find the position of the planet Mars at time t = December 8, 2016, 6:00 pm EST. You will do this by following the steps shown below.

(a) Convert the time t to Universal Time (just add 5 hours to EST).

(b) Find the Julian day corresponding to time t (using the result of part (a)).

(c) Find the time elapsed from the epoch time to time t (i.e., find $t - T_0$).

(d) Find the mean daily motion n (using Kepler's Third Law, Eq. 1). (*Ans.* $n = 1.455689 \times 10^{-3}$ rev/day)

(e) Find the mean anomaly M of Mars at time t (Eq. 2).

(f) Solve Kepler's equation (Eq. 3) to find the eccentric anomaly E of Mars at time t. See Section 3 for information on solving Kepler's Equation.

(g) Find the true anomaly f of Mars at time t (Eq. 4).

(h) Find the distance r of Mars from the Sun at time t (Eq. 5).

(i) Find the argument of latitude u of Mars at time t (Eq. 6).

(j) Find the heliocentric ecliptic cartesian coordinates of Mars at time t (x, y, z) (Eqs. 7–9).

(k) Find the geocentric ecliptic cartesian coordinates of Mars at time t (x_e, y_e, z_e) (Eqs. 10–12).

(l) Find the ecliptic longitude λ and ecliptic latitude β of Mars at time t (Eqs. 13–14).

(m) Find the right ascension α and declination δ of Mars at time t (Eqs. 15–16).

Extra credit:

(n) Find the Greenwich sidereal time GST for time t at Washington D.C. (Eq. 17)

(o) Find the local hour angle H. (Eq. 18)

(p) Find the azimuth A and elevation h of Mars at time t, as seen from Washington D.C. (Eqs. 19–20)

Is Mars above the horizon ($h > 0$)? If so, go outside and see if you can see it at the place you predict it to be.

1 Data

Orbital Elements of Mars (Ecliptic)		
Semi-major axis	a	1.52366231 AU
Eccentricity	e	0.09341233
Longitude of ascending node	Ω	$49^\circ\!.57854$
Inclination	i	$1^\circ\!.85061$
Argument of perihelion	ω	$286^\circ\!.46230$
Mean anomaly at epoch time	M_0	$19^\circ\!.41248$
Epoch time	T_0	JD 2451545.0

Constants		
Astronomical unit	AU	$1.49597870 \times 10^{11}$ m
Obliquity of ecliptic	ε	$23^\circ\!.4392911$
Gravitational constant	$GM_\odot$	$1.32712438 \times 10^{20}$ m^3 s^{-2}
Latitude of Washington	φ	$+38^\circ\!.88$
Longitude of Washington	L	$+77^\circ\!.03$

Sun position at time t (J2000 geocentric ecliptic)	
$x_\odot$	−0.36868482 AU
$y_\odot$	−0.91466548 AU
$z_\odot$	+0.00002696 AU

2 Equations

Mean daily motion n (rev/day), from Kepler's Third Law:

$$n = \frac{86400}{2\pi}\sqrt{\frac{GM_\odot}{a^3}} \tag{1}$$

Mean anomaly M at time t (rad):

$$M = M_0 + 2\pi n(t - T_0) \tag{2}$$

Eccentric anomaly E (Kepler's equation) (rad):

$$M = E - e\sin E \tag{3}$$

True anomaly f (rad):

$$\tan\left(\frac{f}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right) \tag{4}$$

Radial distance r (m):

$$r = a(1 - e\cos E) \tag{5}$$

Argument of latitude u (rad):

$$u = \omega + f \tag{6}$$

Heliocentric cartesian ecliptic coordinates (x, y, z) (m):

$$x = r(\cos u\cos\Omega - \sin u\sin\Omega\cos i) \tag{7}$$
$$y = r(\cos u\sin\Omega + \sin u\cos\Omega\cos i) \tag{8}$$
$$z = r\sin u\sin i \tag{9}$$

Geocentric cartesian ecliptic coordinates (x_e, y_e, z_e) (m):

$$x_e = x + x_\odot \tag{10}$$
$$y_e = y + y_\odot \tag{11}$$
$$z_e = z + z_\odot \tag{12}$$

Geocentric ecliptic longitude λ and ecliptic latitude β (deg):

$$\tan\lambda = \frac{y_e}{x_e} \tag{13}$$
$$\sin\beta = \frac{z_e}{\sqrt{x_e^2 + y_e^2 + z_e^2}} \tag{14}$$

Right ascension α and declination δ (deg):

$$\tan\alpha = \frac{\sin\lambda\cos\varepsilon - \tan\beta\sin\varepsilon}{\cos\lambda} \tag{15}$$
$$\sin\delta = \sin\beta\cos\varepsilon + \cos\beta\sin\varepsilon\sin\lambda \tag{16}$$

Greenwich sidereal time GST (deg):

$$\begin{aligned} GST &= 280.46061837 + 360.98564736629(t - 2451545.0) \\ &+ 0.000387933T^2 - T^3/38710000, \end{aligned} \tag{17}$$

where $T = (t - 2451545.0)/36525$, and t is the Julian day from part (b).

Local hour angle H:

$$H = GST - L - \alpha \tag{18}$$

Azimuth A and elevation h:

$$\tan A = \frac{\sin H}{\cos H \sin\varphi - \tan\delta\cos\varphi} \tag{19}$$

$$\sin h = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos H \tag{20}$$

Here the azimuth A is measured westward from south, so that $A = 0°$ is south, $A = 90°$ is west, $A = 180°$ is north, and $A = 270°$ is east. The elevation h is the angle of Mars above the horizon.

3 Solving Kepler's Equation

Kepler's equation relates the mean anomaly M to the eccentric anomaly E:

$$M = E - e \sin E,$$

where e is the eccentricity of the orbit, and both M and E must be in *radians*. We are given M and e, and wish to solve for E. This cannot be done in closed form, but must be done numerically.

One fairly straightforward method of solution is to use Newton's method, in which we make an initial estimate of E (called E_1), then use that estimate to generate a better estimate E_2. Estimate E_2 is then used to generate an even better estimate E_3, and so forth. In Newton's method, each estimate E_{n+1} of E is found from the previous estimate E_n by

$$E_{n+1} = E_n - \frac{M - E_n + e \sin E_n}{e \cos E_n - 1}$$

You can use M as an initial estimate for E (i.e., use $E_1 = M$).

3.1 Example

Suppose we have an orbit for which $M = 60^\circ\!.00$ and $e = 0.1500$. We first convert M to radians:

$$M = 60^\circ\!.00 \times \frac{\pi}{180} = 1.047198 \text{ rad}$$

Then a few iterations of Newton's method gives

$$\begin{aligned}
E_1 &= M = \boxed{1.047198 \text{ rad}} \\
E_2 &= E_1 - \frac{M - E_1 + e \sin E_1}{e \cos E_1 - 1} \\
&= 1.047198 \text{ rad} - \frac{1.047198 \text{ rad} - 1.047198 \text{ rad} + 0.1500 \sin(1.047198 \text{ rad})}{0.1500 \cos(1.047198 \text{ rad}) - 1} \\
&= \boxed{1.187634 \text{ rad}} \\
E_3 &= E_2 - \frac{M - E_2 + e \sin E_2}{e \cos E_2 - 1} \\
&= 1.187634 \text{ rad} - \frac{1.047198 \text{ rad} - 1.187634 \text{ rad} + 0.1500 \sin(1.187634 \text{ rad})}{0.1500 \cos(1.187634 \text{ rad}) - 1} \\
&= \boxed{1.186243 \text{ rad}} \\
E_4 &= E_3 - \frac{M - E_3 + e \sin E_3}{e \cos E_3 - 1} \\
&= 1.186243 \text{ rad} - \frac{1.047198 \text{ rad} - 1.186243 \text{ rad} + 0.1500 \sin(1.186243 \text{ rad})}{0.1500 \cos(1.186243 \text{ rad}) - 1} \\
&= \boxed{1.186242 \text{ rad}}
\end{aligned}$$

After a few steps, the answer converges to $E = 1.186242$ radians, or $E = 67^\circ\!.9667$.

4 General Notes and Hints

1. Don't wait until the last minute to start this problem! It will take some time. Work carefully; a mistake made early on will affect all the following results.

2. This assignment can be done using only a calculator. You might wish to write a computer program to do it, however, to avoid repeating calculations if you make a mistake. Either way is fine.

3. If you are using a calculator, make sure it is set for "radians" mode when doing calculations in radians. Most computer programming languages work *only* in radians.

4. In Eq. 1, you must convert a from AU to meters before plugging in to the equation. (Likewise for other distances in other equations—convert from AU to meters.)

5. In Eq. 2, n is in rev/day, and t and T_0 are in Julian days.

6. In Eqs. 2–3, the angles *must* be in radians.

7. In Eqs. 2–4, the mean anomaly M, eccentric anomaly E, and true anomaly f should all be approximately equal (within a few degrees), since the orbit is nearly circular. Knowing that should help you check your answers.

8. When calculating the inverse tangent (Eqs. 13, 15, and 19), remember the rule for placing the inverse tangent of a ratio in the correct quadrant: if the denominator is negative, then add 180° to your calculator's answer. Alternatively, you can use your calculator's rectangular-to-polar conversion function ("R ► Pθ" on the TI-83+) to get the inverse tangent in the correct quadrant. If you are writing a computer program, the programming language may have a function like `atan2` that does the same thing. You will get the wrong answer unless you put the inverse tangent in the correct quadrant!

9. It's generally best to reduce all your angles to the range 0–360° (0–2π rad). Remember that you can always add or subtract any number of multiples of 360° (2π rad) to an angle; the result will be equivalent to the original angle.